Re(dis)covering Leibniz's Diagrammatic Logic

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Abstract

In this paper I attempt to retrieve Leibniz's linear diagrammatic logic for his syllogistic and highlight its computational and logical features by providing a formal approach to it in metalogical terms, which is something that, as far as we know, has not yet been accomplished. Thus, in this contribution I pursue two goals, one historical and the other logical: *i)* to bring more attention on the algorithmic aspects of Leibniz's linear diagrammatic system for his syllogistic, which I believe has been neglected because of a general bias against diagram-based reasoning; and *ii)* to prove the metalogical properties of the system in order to argue that such a system is a *bona fide* logical system.

Key words: diagrammatic reasoning, diagrams, linear diagrams, metalogic.

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Redescubriendo la lógica diagramática de Leibniz

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Resumen

En este artículo recuperamos la lógica diagramática lineal de Leibniz para la silogística y descubrimos sus propiedades lógicas y computacionales a través de una aproximación formal en términos metalógicos, lo cual es algo que, hasta donde sabemos, aún falta por hacerse. Así, en esta contribución buscamos, respectivamente, dos metas, una histórica y una lógica: *i)* prestar más atención a los aspectos algorítmicos del sistema diagramático lineal de Leibniz para la silogística, de los cuales creemos que han sido desdeñados por un prejuicio general en contra del razonamiento diagramático; y *ii)* probar propiedades metalógicas del sistema para argumentar que es un sistema lógico *bona fide*.

Palabras clave: razonamiento diagramático, diagramas, diagramas lineales, metalógica.

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1. Introduction

In this paper I retrieve Leibniz's linear diagrammatic logic for his syllogistic as it appears in *De formæ logicæ comprobatione per linearum ductus* (*Phil.*, VII, B, IV, 1-10) (Leibniz & Couturat, 1903: 292-328) and highlight its computational and logical features by providing a revision of it in metalogical terms, which is something that, as far as I know, has not yet been accomplished (Cf. Lenzen, 1990, 2004; Velarde Lombraña, 2002).

Thus, in this contribution I pursue two goals, one historical and the other logical: *i)* to bring more attention on the algorithmic aspects of Leibniz's linear diagrammatic system for his syllogistic (henceforth, *LEIB*), which I believe has been neglected because of a general bias against diagram-based reasoning, as we shall see; and *ii)* to prove the metalogical properties of *LEIB* that give evidence that such a system is a *bona fide* logical system—and not only a didactic tool—which may shed some light on other aspects of Leibniz's philosophy in general and heterogeneous logic in particular.

The paper is organized in the following way. In Section 1 I begin with a brief exposition of the usual concepts of *logic* and *logical system* in order to describe the typical approaches to logical consequence. In Section 2, I develop some ideas that lie behind the notion of *diagrammatic logical consequence*. Later, in Section 3, I explain some of the general aspects of *syllogistic* and *VENN* so as to exemplify the preceding notions and introduce the basic aspects of a diagrammatic logic. After that, I present *LEIB*'s syntax and semantics and show its metalogical attributes in order to argue that *LEIB* is a *bona fide* (diagrammatic) logical system. Finally, I briefly discuss the results of my study and close by giving some pointers concerning future research.

2. Logic, Logical Systems, and Diagrammatic Logical Consequence

2.1 Logic

Reasoning is a process that produces new information given previous data by following certain norms that allow us to describe inference as the unit of measurement of reasoning: inference may be more or less (in) correct depending on the compliance or violation of such norms. The science that studies such norms is *logic*.

The structural understanding of logic has depended, typically but not exclusively, on equivalent sentential approaches: the semantical, the syntactical, and the abstract. From the semantical standpoint the central concept of such norms is that of *interpretation* and defines our notions of *satisfaction*, *model*, and *logical truth* as denoted by $\vDash$ and is usually attributed to Tarski (1956a). From the syntactical point of view the main concept of those norms is that of *deducibility*; it characterizes our intuitions of *proof*, *demonstration*, and *theorem* as denoted by $\vdash$ and is usually attributed to Carnap (1937). Finally, from the abstract standpoint the idea behind those norms is that of a *function of consequence* that generalizes the previous accounts as typically attributed to Tarski (1956b). These approaches are sentential and they define logical consequence as the *proprium* of logic. As an example, consider classical logic: its logical consequence relation follows the next structural norms where and are sentences and is a set of sentences: reflexivity (if $\varphi \in \Gamma$, then $\Gamma \vdash \varphi$), monotonicity (if $\Gamma \vdash \varphi$, then $\Gamma \cup \{\psi\} \vdash \varphi$), and cut (if $\Gamma \vdash \psi$ and $\Gamma \cup \{\psi\} \vdash \varphi$, then $\Gamma \vdash \varphi$).

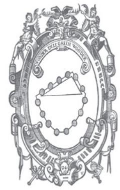
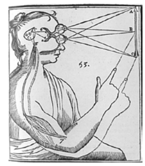
2.2 Logical systems

Logical systems, the tools used to model and better understand the relation of consequence may be defined by a pair $\langle L,B \rangle$ where L is a language, and B is a semantic base (often equivalent to a calculus). Usually, the vocabulary is made up of two sets of signs: variables (non-logical signs) and constants (logical signs). Syntax is used to determine, uniquely and recursively, the well-formed expressions of the system and semantics is used to provide meaning to such well-formed expressions. A well-defined logical system must have these elements. To illustrate this notion of logical system let us consider classical propositional logic, L_0. Its vocabulary has the constants $CONS=\{\neg,\vee\}$ and the variables VAR={φ_1,φ_2,...}. It has two syntax rules: *i)* if $\varphi \in VAR$, then φ is a well-formed formula (wff) of L_0; and *ii)* if φ and ψ are wffs of L_0, then $\neg\varphi$ and $\varphi \vee \psi$ are also wffs of L_0. The semantics of L_0 is composed by a domain and a function of interpretation. The domain of L_0 is the set of truth values, $D=\{1,0\}$, where 1 stands for the designated value and 0 for the anti-designated value. The function of interpretation f maps the variables to the truth values, $f{:}VAR \longrightarrow D$. With this function a valuation v is defined in such way that $v(\varphi)=f(\varphi)$, $v(\neg\varphi)=1\text{-}v(\varphi)$, and $v(\varphi\vee\psi)=max(v(\varphi),v(\psi))$. This valuation provides the rules of correspondence that allow us to build the truth tables of L_0 along with the remaining connectives and tautologies which, due to soundness and completeness, are equivalent to a mechanichal calculus.

2.3 Diagrams and diagrammatic logical consequence

I shall show that diagrammatic logical systems can be defined in a similar fashion and that we can describe a well-behaved notion of diagrammatic logical consequence between diagrams (and not sentences); but before we do that, I would like to introduce diagrams by paying attention to their expressive power.

Pop culture already somehow recognizes this expressive power: the 19th century proverb "a picture is worth 1000 words" is quite representative in this sense; but the basic idea is much older. Notable examples of confidence in the expressive power of diagrams can be found in different historical periods. Ramon Llull (1232-1315), for instance, is arguably the most famous case: he developed his *Ars Magna*, a diagrammatic device used to *explain* the divine nature to those unable to understand God, under the assumption that diagrammatic methods were more convincing or expressive than sentential representations (Figure 1). Thomas Murner (1475-1537) also used diagrams in his *Logica Memorativa* in order to *teach* logic (Figure 2). Dutch mathematician and philosopher of science Simon Stevin (1548-1620) developed another remarkable diagram in his *demonstration* that the efficiency of the inclined plane is a logical consequence of the impossibility of perpetual motion (Figure 3). And, of course, besides these examples we have Descartes' (1596-1650), who is perhaps the most famous case of a modern philosopher who made good use of diagrams in order to *model* hypotheses, such as the mechanics of the pineal gland (Figure 4).

			
Figure 1. A diagram from Llull's *Ars Magna* (Llull, 1501)	**Figure 2.** A diagram from Murner's *Logica Memorativa* (Murner, 1967)	**Figure 3.** A diagram from Stevin's *De weedgaet* (Stevin, 1586)	**Figure 4.** A diagram from Descartes' *Principles of Philosophy* (Descartes, Miller & Miller, 1984)

Our point is that the expressive power of diagrams for aiding *reasoning* was not news for modern thinkers and Leibniz was no exception: we can find evidence for this statement in an 18th century research program for diagrammatic reasoning in the work of Leibniz (Figure 5), Lambert (Figure 6), and Plouquet (Figure 7).

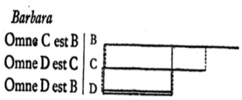
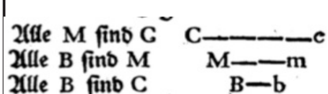
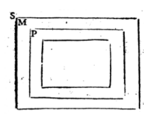
		

Figure 5. A *Barbara* syllogism in *De formæ logicæ comprobatione per linearum ductus* (Leibniz & Couturat, 1903)	**Figure 6.** A *Barbara* syllogism in *Neues Organon* (Lambert, 1764)	**Figure 7.** A *Barbara* syllogism in *Sammlung der Schriften* (Bök, 1766)

This confidence in the expressive power of diagrams is understandable. In order to represent knowledge we use internal and external representations. Internal representations convey mental images, for example; whereas external representations include physical objects on paper, on blackboards, or computer screens. External representations can be divided into two classes: sentential and diagrammatic (Larkin & Simon, 1987).

Sentential representations are sequences of sentences in a particular language. Diagrammatic representations are sequences of diagrams that contain information stored at one particular *locus* in a configuration, including information about relations with the adjacent *loci*; and *diagrams* are information graphics[1] that index information by location on a plane (Larkin & Simon, 1987). The difference between diagrammatic and sentential representations, thus, is that the former preserve explicit information about topological relations, while the latter do not—they may, of course, preserve other kinds of relations.

This confidence in the power of diagrams is comprehensible due to their computational advantages: they group together information avoiding large amounts of text, they automatically support a large number of perceptual inferences—which are extremely easy to interpret—, and they grant the possibility of applying operational constraints (like *free rides* and *overdetermined alternatives* (Shimojima, 1996)) to allow the automation of perceptual inference (Larkin & Simon, 1987).

However, despite this general confidence, when it comes to reasoning there is a bias (or a tradition) that supports the claim that while proof-based reasoning is essential in logic and mathematics, diagram-based reasoning, no matter how useful (Nelsen, 1993) or elegant (Polster, 2004), is not, for it is not *bona fide* reasoning. Thus, for example, Tennant has suggested a diagram is only a heuristic to prompt certain trains of inference (Tennant, 1986); Dieudonné has urged a strict adherence to axiomatic methods with no appeal to geometric intuition, at least in

[1] Information graphics can be divided into the next classes (Nakatsu, 2009): quantitative charts (bar-column charts, line graphs, XY scatterplots, pie charts), maps (directional maps, topographic maps, contour maps, weather maps), tables (one way tables, two ways tables, multiway tables), pictorial illustrations, and diagrams, which we can use to study system topology (conceptual models, network diagrams), sequence and flow (flowcharts, activity diagrams), hierarchy-classification (organization charts, classification hierarchies, composition models), association (semantic networks, entity relationship diagrams), cause and effect (directed graphs, fishbone diagrams, fault tree analysis diagrams), and reasoning (argument diagrams, Euler diagrams, Venn diagrams).

formal proofs (Dieudonné, 2008); Lagrange remarked in the *Preface* to the first edition of his *Mécanique Analytique* (1788) that no figures were to be found in his work (Boissonnade, Lagrange, & Vagliente, 2013); and even Leibniz himself shared a similar opinion at some point (emphasis mine):

> *La force de la démonstration est indepéndante de la figure tracée, qui n'est que pour faciliter l'intelligence de ce qu'on veut dire et fixer l'attention;* ce sont les propositions universelles, c'est-à-dire les définitions, les axiomes et les théorèmes déjà démontrés qui font le raisonnement et le soutiendraient quand la figure n'y serait pas (Leibniz, 1966: 309).

This bias against diagram-based reasoning is based upon the assumption that diagrams naturally lead to fallacies, mistakes, and are not susceptible of generalization. Nevertheless, in retrospect we can find an argument against this assumption in Newton's *Preface* to the *First Edition of Principia Mathematica* (Newton, 1974) by reducing proof-based reasoning to mechanical reasoning (emphasis mine):

> But as artificers do not work with perfect accuracy, it comes to pass that mechanics is so distinguished from geometry that what is perfectly accurate is called geometrical; what is less so, is called mechanical. However, *the errors are not in the art, but in the artificers*. He that works with less accuracy is an imperfect mechanic; and if any could work with perfect accuracy, he would be the most perfect mechanic of all, for the description of right lines and circles, upon which geometry is founded, belongs to mechanics (Newton, 1974: 11).

Along similar lines, Allwein, Barwise, and Etchemendy (1996) and Shin (1994) have developed a successful research program around heterogeneous and diagrammatic reasoning that has promoted different studies and model theoretical schemes that help us represent and better understand diagrammatic reasoning in logical terms, thus allowing a well-defined notion of a diagrammatic logical system and a diagrammatic inference.

Thus if, as we stated above, reasoning is a process that produces new information given previous data, and information can be represented diagrammatically, it is not unreasonable to suggest that diagrammatic inference is the unit of measure of diagrammatic reasoning: diagrammatic inference would be correct or incorrect depending on the compliance or violation of certain norms. Let us denote the relation of diagrammatic logical consequence or diagrammatic inference by $\mapsto$; this relation would define our intuitions around the informal notions of *visual inference* or *visual argument* and would follow, in principle, the standard structural norms where δ is a particular diagram and Δ is a set of diagrams: reflexivity (if $\delta \in \Delta$, then $\Delta \mapsto \delta$), monotonicity (if $\Delta \mapsto \delta$, then $\Delta \cup \{\delta'\} \mapsto \delta$), and cut (if $\Delta \mapsto \delta$ and $\cup\{\delta\} \mapsto \delta'$, then $\Delta \mapsto \delta'$).

To give a somewhat detailed account of these structural properties we need to describe how the operator $\mapsto$ works. Shimojima has developed a logical theory around diagrammatic inference that defines (informally) a *free ride* as a process in which some reasoner gains information without following any step specifically designed to gain it (Shimojima, 1996: 32), in other words, a free ride is a process that allows us to reach automatically (and sometimes inadvertently) a conclusion from a diagrammatic representation of the premises. Inversely, an *overdetermined alternative* occurs when a diagram that should not follow from a given diagrammatic configuration of the premises does follow.

Using Shimojima's approach we could say that *reflexivity* establishes that a if a diagram is part of a set of diagrams or diagrammatic configuration, then that particular diagram is a visual consequence of such configuration because there is a free ride from the diagrammatic configuration to a particular diagram; *monotonicity* would say that if a diagram is a free ride from a diagrammatic configuration, and a new diagram is added to such configuration, then the initial diagram is still a free ride from the original configuration.[2] Finally, *cut* would establish that if a diagram is a free ride from a diagrammatic configuration and

[2] In other place we have argued that monotonicity is not a *bona fide* property of diagrammatic inference, but for the purposes of this paper the assumption of monotonicity will suffice.

the addition of a new diagram produces a new free ride, there is a free ride from the original diagrammatic configuration to the new diagram.

3. Syllogistic and *VENN*

The previous description may seem too abstract for something as concrete as a diagram. So, in this section I exemplify the preceding notions and introduce the basic aspects of a diagrammatic logical system by using syllogistic and *VENN*. I use syllogistic for two obvious reasons: first, because syllogistic is a paradigmatic mode of reasoning given that it is relevant to us in many ways. Logically, as a basis for science. Historically, as a tradition that gathers ancient and contemporary logicians. And pedagogically, as a trademark of our education and culture: most undergraduate logic courses, books, and manuals around the world cover or include a fragment of syllogistic not just to teach or introduce logic, but to provide *formæ mentis* for scientific reasoning and critical thinking. And secondly, because syllogistic, the object of study in this paper, was quite appreciated by Leibniz (1966) (emphasis mine):

> Je tiens que l'invention de la forme des syllogismes est une des plus belles de l'esprit humain, et même des plus considérables. *C'est une espèce de mathématique universelle, dont l'importance n'est pas assez connue*; et l'on peut dire qu'un art d'infaillibilité y est contenu, pourvu qu'on sache et qu'on puisse s'en bien servir, ce qui n'est pas toujour permis (Leibniz, 1966: 428).

And we use *VENN* not only because it is a very powerful system capable of representing set theoretical assumptions, but also because it is probably the diagrammatic system most used to represent syllogistic.

3.1 Syllogistic

Syllogistic has its origin in Aristotle's *Prior Analytics* and is the theory of inference that deals with the consequence relation between two categorical propositions taken as premises and another categorical proposition taken as a conclusion. A *categorical proposition* is a proposition composed by two terms, a quantity, and a quality. The subject and the predicate of a proposition are called terms: the term-schemadenotes the subject term of the proposition and the term-schema *P* denotes the predicate. The quantity may be either universal (*All*) or particular (*Some*) and the quality may be either affirmative or negative. These categorical propositions are denoted by a label, either *A* (universal affirmative), *E* (universal negative), *I* (particular affirmative), or *O* (particular negative). A *categorical syllogism*, then, is a sequence of three categorical propositions ordered in such a way that two propositions are premises and the last one is a conclusion. Within the premises there is a term that appears in both premises but not in the conclusion. This particular term works as a link between the remaining terms and is known as the middle term, which we will denote with the term-schema *M*. According to this term we can set up four figures that encode and abbreviate all the valid and only the valid syllogisms (Table 2).

Figure 1	Figure 2	Figure 3	Figure 4
Barbara *MAPSAM⊢SAP*	*Cesare* *PEMSAM⊢SEP*	*Disamis* *MIPMAS⊢SIP*	*C a l e m e s* *PAMMES⊢SEP*
Celarent *MEPSAM⊢SEP*	*Camestres* *PAMSEM⊢SEP*	*Datisi* *MAPMIS⊢SIP*	*Dimaris* *PIMMAS⊢SIP*
Darii *MAPSIM⊢SIP*	*Festino* *PEMSIM⊢SOP*	*Bocardo* *MOPMAS⊢SOP*	*Fresison* *PEMMIS⊢SOP*
Ferio *MEPSIM⊢SOP*	*Baroco* *PAMSOM⊢SOP*	*Ferison* *MEPMIS⊢SOP*	-

Table 2. Valid syllogisms

3.2 VENN

VENN (Venn, 1880) is a sound and complete diagrammatic logical system (Shin, 1994) that represents syllogistic perspicuously. *VENN* can be defined as a (diagrammatic) logical system with a well-defined vocabulary, syntax, and semantics (Shin, 1994: 48). Briefly, the vocabulary is determined by the next elements: the closed curve, the rectangle, the shading, the X, and the line (Figure 8).

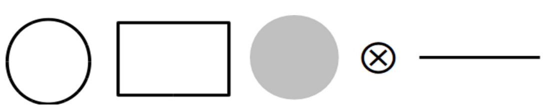


Figure 8. Vocabulary of *VENN*

With this vocabulary a *diagram* in *VENN* is defined as any finite combination of diagrammatic elements (Shin, 1994). A *region* is any enclosed area in a diagram. A *basic region* is a region enclosed by a rectangle or a closed curve. A *minimal region* is a region within which no other region is enclosed. An *X-sequence* is a diagram of alternating X's and lines with an X in each extremal position. Regions represent sets and the rectangle represents the domain. A shaded region represents an empty region and a region with an X represents a non-empty region. And with this syntax the categorical propositions can be represented as follows (Figure 9):

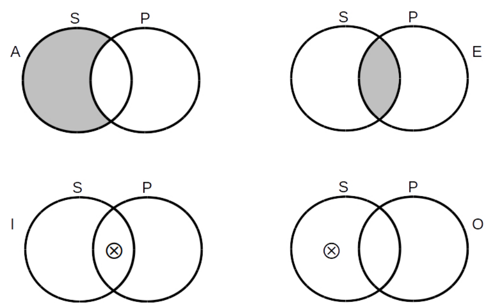


Figure 9. Categorical propositions in *VENN*

Further, the semantics of *VENN* depends on a natural homomorphism with sets that help define the rules for developing (diagrammatic) proofs in this diagrammatic system (Shin, 1994 81):

1. The *rule of erasure of a diagrammatic object* tells us that a well-formed diagram (wfd) *D′* is obtained from a wfd *D* if *D′* results from either erasing a closed curve of *D*, or erasing a shading of some region of *D*, or erasing an entire X-sequence of *D*.

2. The *rule of erasure of parts of an X-sequence* says that is obtained from if it results fromby the erasure of parts in some X-sequence that fall in shaded regions, provided that the remaining X's are reconnected.

3. The *rule of spreading of an X-sequence* tells us that is obtained from if extra X-sequences have been added to some X-sequence of *D*.

4. The *rule of introduction of a basic region* indicates that a basic region may be introduced by drawing either a rectangle or a closed curve.

5. The *rule of conflicting information* says that if a diagram has a region with both a shading and an X-sequence, then we may transform the given diagram into any diagram.

6. The *rule of unification of diagrams* says that *D′* is obtained from two wfds D_1 and D_2 if every region of *D* is a counterpart region of either D_1 or D_2 and conversely. If any region of *D* is shaded or has an X-sequence, then it has a counterpart in either D_1 or D_2 which is also shaded or has an X-sequence and conversely.

The previous rules may be summarized into the rules of *erasure*, *addition*, and *unification* (Nakatsu, 2009: 133). As an example, consider a diagrammatic proof of a syllogism of the form (Figure 10). According to the previous rules we begin with an introduction of areas (step 1) and then a unification is applied (step 2). After that, we apply an erasure of an X-sequence (step 3) and then a spreading of an X-sequence (step 4). Finally, by the erasure of a closed curve rule, we obtain a final diagram (step 5). Since the conclusion got drawn by drawing down the premises, the inference is valid (and corresponds to a *Darii* syllogism).

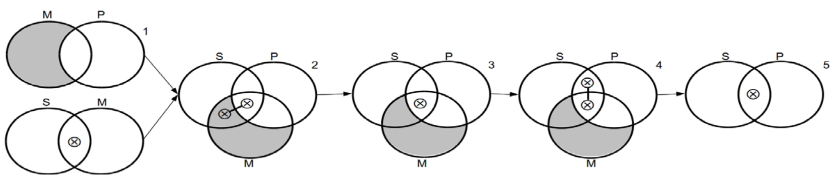


Figure 10. Example of a proof in *VENN*

As we can see, with these rules *VENN* provides an essential feature of a well-defined diagrammatic logic: a linear time diagrammatic method of decision for syllogistic that consists in drawing the diagrams for the premises and then checking (by mere observation) whether it is possible to "read off" the conclusion from the drawing of the premises; in case it does, the syllogism is valid; otherwise it is invalid.

4. Leibniz's linear diagrammatic logic

After this lengthy presentation we are now in a good position to introduce and reconstruct Leibniz's linear diagrammatic logic, *LEIB*, by recovering it as a logical system and by discovering its logical attributes.

The diagrammatic system we are interested in appears in *De formæ logicæ comprobatione per linearum ductus* (*Phil.*, VII, B, IV, 1-10) and we use the edition of Couturat (Leibniz & Couturat, 1903, 292-328).

4.1 LEIB as a logical system

My first goal is to present *LEIB* by reconstructing its vocabulary, syntax, and semantics. In order to do that I pay special attention to *LEIB'*s diagrams for categorical propositions (Figure 11):

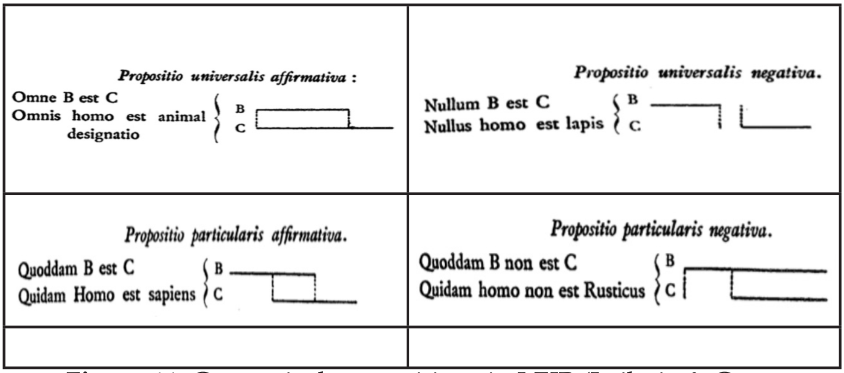


Figure 11. Categorical propositions in LEIB (Leibniz & Couturat, 1903, 292-293)

From these diagrams we can infer that the *vocabulary* of *LEIB* has the following basic diagrammatic elements: the solid horizontal line and the dotted vertical line (Figure 12).

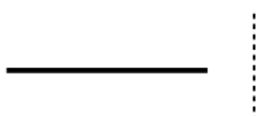


Figure 12. Vocabulary of *LEIB*

With this vocabulary we can define the *syntax* of the wfds for *LEIB*. The solid horizontal lines stand for terms and the vertical lines stand for a relation between terms. Given two horizontal lines representing terms, one could be completely included in another; they could be completely disjoint; they could partially intersect each other; or one could be partially not included in another (Figure 13).

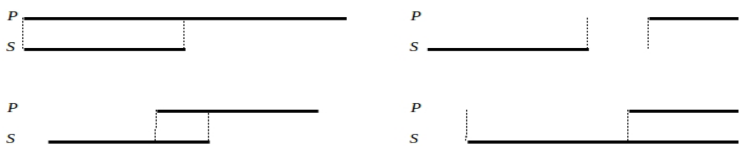


Figure 13. Syntax of *LEIB*

The *semantics* of these wfds is straightforward: a diagram of a proposition *A* shows that all that is *S* is indexed in *P*, but not inversely. Proposition *E* shows that no *S* is indexed in *P* and vice versa. Proposition *I* represents the fact that some *S* is indexed in some *P*, and vice versa. Proposition *O* states that some *S* is not indexed in all *O* (Figure 14).

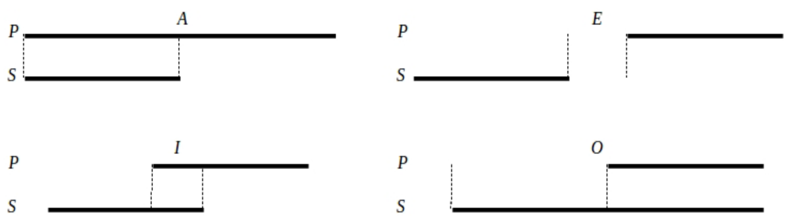


Figure 14. Reconstruction of categorical propositions in *LEIB*

To exemplify how *LEIB* works let us represent a syllogism in a diagrammatic fashion. Figure 15 shows what a *Barbara* syllogism looks like:

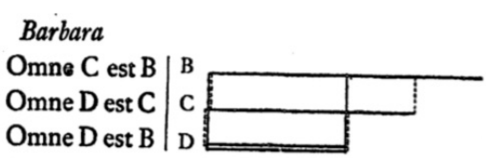


Figure 15. A *Barbara* syllogism as it originally appears in Leibniz & Couturat (1903: 294)

The construction of the previous syllogism is relatively simple. We produce the diagrams that stand for the premises of a *Barbara* syllogism, namely, *MAPSAM*. In order to do that we draw the proposition *MAP* (Figure 16). Then, since *SAM* shares *M* with *MAP*, *SAM* must be drawn with respect to *M* (Figure 17):

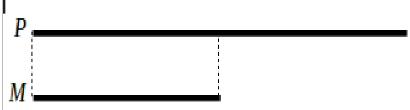


Figure 16. *MAP*

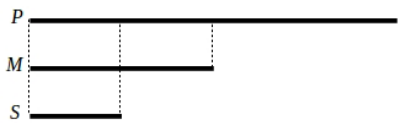


Figure 17. *SAM*

Hence, following the reconstruction, a *Barbara* syllogism would look like the following diagram:

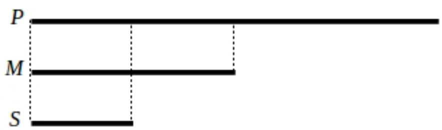


Figure 18. A *Barbara* syllogism reconstructed in *LEIB*

I shall show what canonical syllogisms look like in *LEIB*, but before I do that I would like to focus on the most interesting aspect about *LEIB*: a linear time diagram-based algorithm of decision, call it A, that takes any syllogism as an input and provides a decision about the (in)validity of such syllogism by checking whether the wfd of the conclusion is automatically represented by representing the wfds of the premises (otherwise, the syllogism is invalid) (Table 3).

A (σ)

Input: syllogism σ
If *Prem*(s)↦*Conc*(s)
σ ← *valid*
else
σ ← *invalid*
endIf

Table 3. *LEIB'*s algorithm of decision

Prem takes a syllogism σ and produces a diagram for the premises; ↦ stands for a free ride; and *Conc* checks the diagram of the conclusion of σ. In Figure 19 it is easy to see how the conclusion was automatically obtained by representing the premises, i.e., the conclusion "got drawn" automatically by "drawing down" the premises: this is a fair example of a free ride.

Figure 19. Free ride in a *Barbara* syllogism in *LEIB*

Using the previous decision algorithm and our reconstruction of *LEIB* we can show diagrammatic proof of the valid syllogisms in *LEIB* (Figures 20-23):

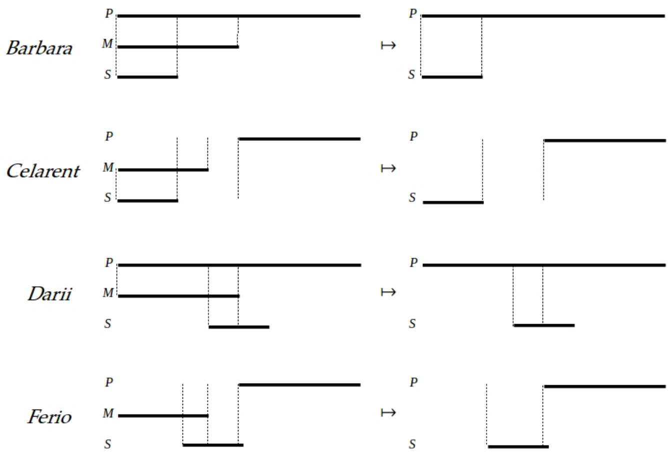


Figure 20. Valid syllogisms from figure 1

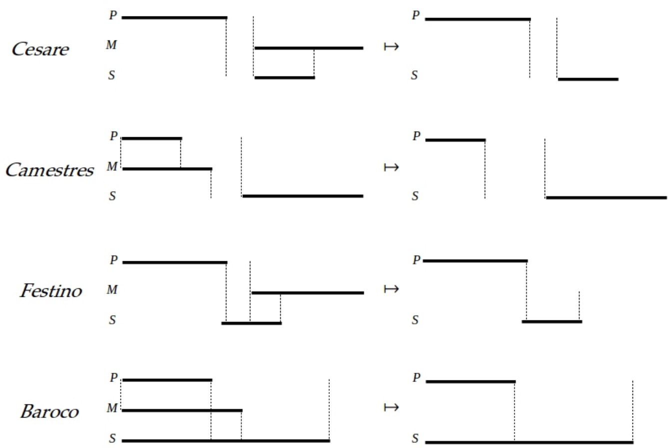


Figure 21. Valid syllogisms from figure 2

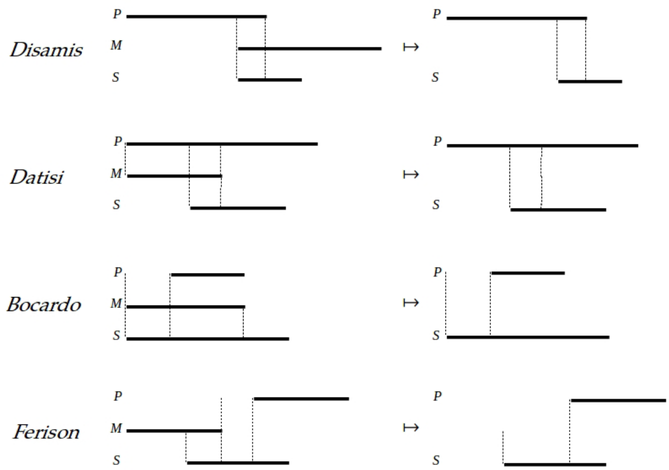


Figure 22. Valid syllogisms from figure 3

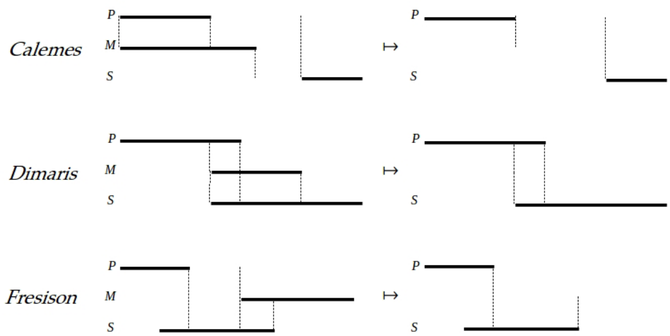


Figure 23. Valid syllogisms from figure 4

Finally, to provide a more comprehensive explanation of how *LEIB* works I would like to show a couple of examples of invalid syllogistic forms. First consider a form with premises *MEPSEM*, which should be invalid. *LEIB* shows that such syllogism is actually invalid because the conclusion is not a free ride, but an overdetermined alternative (because the diagram of the conclusion is not even a wfd) (Figure 24).

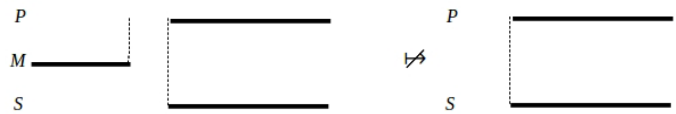


Figure 24. An invalid syllogistic form in *LEIB*

In Leibniz's own account, a form with premises would also be invalid because the conclusion would also be an overdetermined alternative (Figure 25).

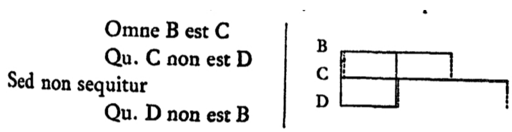


Figure 25. An invalid syllogistic form in Leibniz & Couturat (1903: 299)

4.2 LEIB's logical attributes

After this exploration of *LEIB* as a logical system I would like to pursue my second goal by suggesting a proof for a set of propositions that cover attributes of soundness, completeness, decidability, autarchy, and some sort of equivalence with *VENN* as well as a subset of *BOOL* (Boole, 1951). But before we look at those properties we need a preliminary result that I like to call *Aristotle's Lemma*.

In *Prior Analytics* A.1, 25b1 Aristotle argued that it is possible to reduce all valid syllogisms to the universal syllogisms from figure 1, that is to say, to *Barbara* and *Celarent*. The idea in this proposition is that all valid syllogisms can be reduced, by following some precise instructions (like the ones depicted in Table 4), to other valid syllogisms. This is precisely the phenomenon captured by the medieval tradition of using names as abbreviations as we saw in Table 2 and is also Leibniz's *desideratum*.

Letter	Instruction
B	Reduce syllogism to *Barbara*
C	Reduce syllogism to *Celarent*
D	Reduce syllogism to *Darii*
F	Reduce syllogism to *Ferio*
S	Apply conversion to a proposition
P	Apply conversion *per accidens* to a proposition
M	Move places of major and minor premises
C	Apply a contradiction

Table 4. Instructions for the reduction of syllogisms

Hence, for example, a *Cesare* syllogism can be reduced to a *Celarent* syllogism because "Cesare" starts with letter *C*, and this is possible by a conversion of the proposition *E*, which is indicated by the letter *s* right after the letter *e* that stands for a categorical proposition *E*. We can see that this idea holds in *LEIB*.

Proposition 1. (Aristotle's lemma for *LEIB*) Every valid syllogism is reducible to some syllogism from figure 1.

Proof. We prove this diagrammatically with the aid of some rigid motions (Figures 25-27). In Figure 25 we can see *Cesare* and *Celarent* preserve the same diagram by reflection w.r.t. a *Y* axis; *Camestres* and *Celarent* preserve the same diagram by applying a 180° rotation and then a reflection w.r.t. a *Y* axis. *Festino* and *Ferio* preserve the same diagram by reflection w.r.t. the *Y* axis. *Baroco* is reduced to *Barbara* by applying a contradiction of the conclusion using it as a premise.

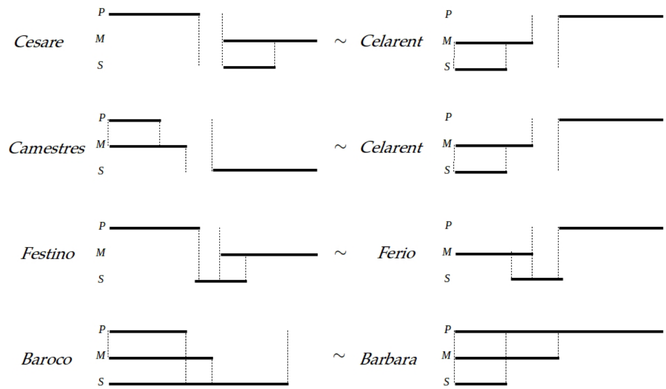


Figure 25. Reduction of syllogisms from figure 2 to syllogisms from figure 1

In Figure 26 we can see *Disamis* and *Darii* preserve the same diagram by applying a 180° rotation; *Datisi* and *Darii*, and *Ferison* and *Ferio* preserve the same diagram. *Bocardo* is reduced to *Barbara* by contradiction.

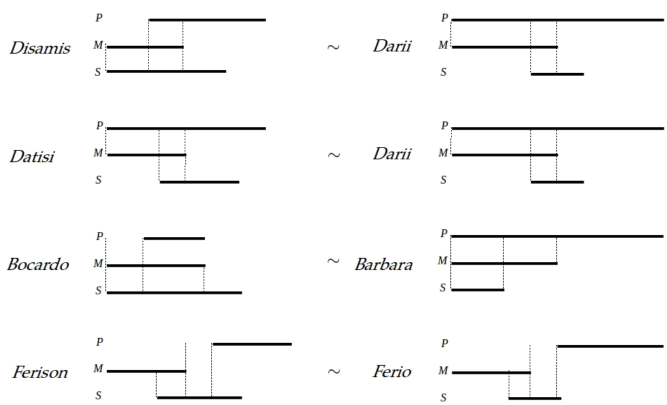


Figure 26. Reduction of syllogisms from figure 3 to syllogisms from figure 1

Finally, in Figure 27 we can see *Calemes* and *Celarent* preserve the same diagram by applying a 180° rotation and then a reflection w.r.t. a

Y axis. *Dimaris* and *Darii* preserve the same diagram by applying a 180° rotation. *Fresison* is reduced to *Ferio* by reflection w.r.t. a *Y* axis.

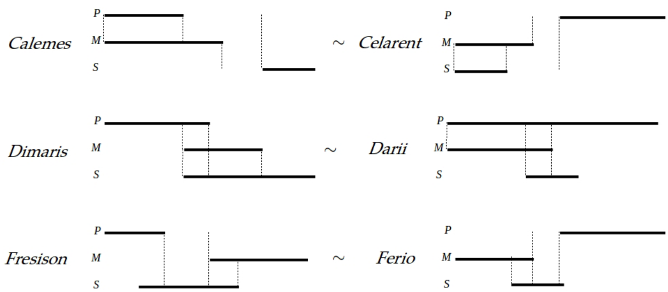


Figure 27. Reduction of syllogisms from figure 4 to syllogisms from figure 1

With this result we can proceed to show that *LEIB*'s algorithm is sound and complete. Let us denote the application of A to a given syllogism $\sigma_{i,j}$ from figure *i*∈{1,2,3,4} and row *j*∈{1,2,3,4} in Table 2 by A (thus, for instance, the application of A to a *Dimaris* syllogism is A ($\sigma_{4,2}$) and, for sake of exposition, A ($\sigma_{4,4}$) is a placeholder).

Proposition 2. (Soundness) If A (σ)=*valid*, then σ is valid.

Proof. We prove this proposition by cases. Since there are four figures, we need to cover each valid syllogism from each figure, which is precisely what we have done in the previous section (Figures 20-23). Thus, we have that for every $\sigma_{i,j}$, when A($\sigma_{i,j}$)=*valid*, $\sigma_{i,j}$ is valid.

Proposition 3. (Completeness) If σ is valid, then A (σ)=*valid*.

Proof. We prove this by contradiction. Suppose that for all *i,j*, the syllogism $\sigma_{i,j}$ is valid but for some valid syllogism $\sigma_{k,j}$, A($\sigma_{k,j}$)=*invalid*. Now, we know $\sigma_{1,j}$ is valid and if we apply A($\sigma_{1,j}$) we obtain A($\sigma_{1,j}$)=*invalid*, as we can see from Proposition 2. Now, since all valid syllogisms $\sigma_{n>1,j}$ can be reduced to the valid syllogisms from figure 1 by Proposition 1 (Aristotle's lemma), it follows that A($\sigma_{n>1,j}$)=*valid*, and thus, for all valid syllogisms *k*,A ($\sigma_{k,j}$)=*valid*, which contradicts our initial assumption.

From these results it follows that a syllogism σ is valid if and only if A (σ)=*valid*, and since A is a diagrammatic decision procedure we can infer, as a corollary, that:

Proposition 4. (Decidability) *LEIB* is decidable.

Indeed, algorithm A is a mechanical and diagrammatic procedure that is sound and complete; but perhaps the importance of these results lies in their relation to two important properties: autarchy and some sort of equivalence with *VENN* using three sets or regions, call it $VENN_3$.

The *autarchy* of a diagrammatic system corresponds to a trade-off between free rides and overdetermined alternatives, that is, a compromise between valid and invalid diagrams (i.e., diagrams that do not follow from the configuration of the premises). An autarchic diagrammatic system is, thus, a system with a set of operational constraints that always give rise to free rides and never to overdetermined alternatives (Bellucci, Moktefi & Pietarinen, 2013). Since *LEIB* is sound, complete, and decidable, it follows that:

Proposition 5. (Autarchy) *LEIB* is autarchic.

This is an important result that will have impact on Leibniz's own account of *LEIB*, as we shall see in Section 4.3; specially because:

Proposition 6. (Equivalence) *LEIB* is equivalent to $VENN_3$ w.r.t. syllogistic.

Proof. In order to provide proof for this statement we show that every valid syllogism in *LEIB* (say, theorem of *LEIB*) is a valid syllogism in $VENN_3$ (say, theorem of $VENN_3$) and vice versa. From left to right: suppose that for any valid syllogisms , is a valid syllogism in *LEIB* but an invalid one in $VENN_3$. Given the soundness and completeness of *LEIB*, being $\sigma_{i,j}$ a valid syllogism in *LEIB* implies that $\sigma_{i,j}$ is valid *simpliciter*. But since *VENN* is sound and complete as well (Shin, 1994), if $\sigma_{i,j}$ is invalid in $VENN_3$, then $\sigma_{i,j}$ must be invalid, which is a contradiction. From right to left the proof is similar.

What this brief metalogical exploration shows is that *LEIB* is not only a logical system, but an actual *bona fide* diagram-based logic because it produces valid (soundness) and only valid inferences (completeness) by providing a time efficient ($O(n)$) mechanical method of decision (decidability) that helps the automation of perceptual inference (autarchy), while preserving equivalence with *VENN*$_3$ (w.r.t. syllogistic) and, thus, with some subset of *BOOL* (equivalence).

4.3 Leibniz vs LEIB

After these results we would like to introduce an interesting puzzle that originates from a conflict between the previous formal account of *LEIB* and Leibniz's own account. Consider that, by Proposition 6, *LEIB* is equivalent to *VENN*$_3$ w.r.t. syllogistic and thus, it is equivalent to some subset of *BOOL*. This fact is important because it means that *LEIB* does not allow us to infer particular propositions from universal propositions due to issues with empty classes: in fact if, for instance, we try to get a *Barbari* syllogism in *LEIB* we will find that a such task is impossible and any attempt to build a *Barbari* necessarily yields a *Barbara* syllogism, due to autarchy, and the same happens with other syllogisms that require existential import.

In the opinion of Kneale and Kneale (1962), Leibniz was by no means an Aristotelian purist, but he was committed to the assumption of existential import (p. 322). Indeed, Leibniz thought that his diagrammatic system was capable of modeling and representing syllogisms with existential import such as *Barbari*, *Cesaro*, *Fesapmo*, *Calemos*, and so forth, which implies the possibility of deriving particular propositions from universal ones (*Phil.*, IV, 50; *Math.*, V, 27; Cf. *New essays*, IV, XVII, 4). Nevertheless, it is not clear how this is possible since *LEIB* is autarchic and equivalent to *VENN*$_3$ and *BOOL*.

The previous situation leads to an interesting puzzle between consistency and eligibility: if Leibniz proposed his system only as a representation system, then it would be a trivial system because it would represent any syllogism, both valid and invalid; but it is quite clear that Leibniz himself argued against such a thesis. Therefore, his system was proposed rather as a non-trivial reasoning system, but then, by Propositions 1-6, by being equivalent to *VENN*$_3$ and *BOOL*, it has a model in a modern interpretation of syllogistic that assumes empty terms. The puzzle is, thus: why would Leibniz argue that his system is capable of modeling such imperfect syllogisms (implying the acceptance of empty terms) when it actually does not (implying the rejection of empty terms)? I believe the answer to this puzzle is beyond the logical scope, and so, we leave the question open.

5. Conclusions

Today the typical diagrammatic treatments of syllogistic include Venn-Euler diagrams (Venn, 1880), Carroll's triliteral diagrams (Carroll, 1887), Karnaugh maps (Karnaugh, 1953), and more recently, Pagnan's SYLL (Pagnan, 2012). In this paper I have introduced another diagrammatic treatment of syllogistic by re(dis)covering Leibniz's diagrammatic logic, which should be an interesting task historically and logically because this diagrammatic system has not been explored before in metalogical terms and because we have showed (our reconstruction of) *LEIB* has a sound and complete algorithm for syllogistic that provides more evidence on the thesis that diagrammatic reasoning is *bona fide* reasoning.

In sum, our results *i)* show formal evidence that *LEIB* is closer to the logical interpretations of *BOOL* (Boole, 1951) and *VENN* (Venn, 1880) than to the medieval tradition, which is not a novelty, but is interesting nevertheless; *ii)* provide a formal approach to Leibniz's linear diagrammatic logic in metalogical terms, which is something that, as far as I know, has not yet been accomplished; and *iii)* offer some sort of informal support for the thesis that some fragment of Leibniz's *mathesis* has modern ontological commitments.

Finally, as part of our current and future work, I would like to add that I am developing similar metalogical reconstructions for other diagrammatic logical systems (old and new, traditional and original) in order to promote the study of mechanical and diagram-based reasoning as a research program with applications, mainly, in philosophy and Artificial Intelligence. In the meantime, I would like to reconsider Leibniz' famous passage (emphasis mine):

> L'unique moyen de redresser nos raisonnemens est de les rendre aussi sensibles que le sont ceux de Mathematiciens, en sorte qu'on puisse trouver son erreur *à veue d'oeil,* | et quand il ya des disputes entre les gens, on puisse dire suelement: *contons*, sans autre ceremonie, pour *voir* lequel a raison (Leibniz & Couturat, 1903, 176).

And rewrite those insightful last words in order to restate: let us *draw diagrams*, without further ado, to *see* who is right!

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