The Sorites Meets the Many[*]

Ricardo Mena
IIF-UNAM
ricardo.mena21@gmail.com

Abstract

The objective of this paper is to understand certain issues that come up once we recognize that a good number of natural language predicates are indeterminate in two different ways. For example, the predicate 'is a mountain' is both vague and susceptible to the problem of the many. Throughout the paper I focus on how to distinguish these two kinds of indeterminacy, and on a certain problem that supervaluationism has when it is recognized that a single predicate can be vague and susceptible to the problem of the many. The problem is that supervaluationism loses its ability to capture our intuitions concerning sharp cut-offs. Finally, I offer a solution to this problem.

Key words: philosophy of language, philosophy of logic, vagueness, the problem of the many, semantic indeterminacy, supervaluationism.

Resumen

El objetivo de este artículo es entender ciertas cuestiones que surgen cuando reconocemos que un buen número de predicados del lenguaje natural son indeterminados de dos maneras diferentes. Por ejemplo, el predicado 'es una montaña' es vago y susceptible al problema de los muchos. A lo largo de este artículo me enfocaré en cómo distinguir estos dos tipos de indeterminación, y en un problema que el supervaluacionismo

Recibido: 11 - 08 - 2014. Aceptado: 16 - 09 - 2014.

This is a shortened version of an unpublished paper that I wrote as a second year graduate student. I was more optimistic about the prospects of supervaluationism back then. However, I still think that it is useful to think about certain issues within the supervaluationist framework.

tiene cuando reconoce que un predicado puede ser vago y susceptible al problema de los muchos. La dificultad es que el supervaluacionismo pierde su habilidad para capturar nuestras intuiciones acerca de cortes precisos. Finalmente, ofrezco una solución a dicho problema.

Palabras clave: filosofía del lenguaje, filosofía de la lógica, vaguedad, el problema de los muchos, indeterminación semántica, supervaluacionismo.

1. Introduction

Indeterminacy is an extensive phenomenon. Predicates, names, quantifiers, and sentences can be, and often are, indeterminate in some respects. There are clear cases of bald people, clear cases a non-bald people, and cases in which it is not clear at all whether the person in question is bald or not. Most English predicates are like 'bald' in this respect. Likewise, proper names can be indeterminate; there are particles that are clearly part of Kilimanjaro, particles that are clearly not part of Kilimanjaro, and particles such that it is not altogether clear whether they are part of Kilimanjaro. Thus, it cannot be completely clear what body of particles 'Kilimanjaro' refers to. Most names are like 'Kilimanjaro' in this respect. These are just two examples among many.

Throughout this paper I shall assume that indeterminacy is a semantic phenomenon, rather than an epistemic (Williamson,1994; Sorensen, 2001) or a metaphysical one (Barnes, 2010). Also, for the sake of making the discussion more manageable, I shall focus exclusively on the supervaluationist (Fine, 1975; McGee and McLaughlin, 1995) perspective on indeterminacy. This is, by and large, the most popular approach to indeterminacy. This view faces some difficulties, but it is nevertheless fruitful to think in its framework when tackling certain issues.

Now, semantic indeterminacy has many shapes. Vagueness doesn't exhaust all of them. For instance, it can be indeterminate whether a predicate applies to certain objects without this predicate being vague. This is so when there is a sharp cut-off between the cases in which the predicate clearly applies and cases in which it is indeterminate whether the predicate applies (and between the cases in which the predicate clearly doesn't apply and cases in which this is indeterminate). [1] Not only names, but predicates as well, can be indeterminate in the sense that it can be indeterminate whether the name refers to one among many objects (Quine, 1960). Quantifiers can be indeterminate as well, in the sense that it can be indeterminate what their domain of quantification is (Lappin, 2000). Even if the source of all kinds of indeterminacy is semantic, it is interesting to study its many shapes.

The focus of this paper shall be on two kinds of predicate indeterminacy: sorites and selection indeterminacy. By *sorites indeterminacy* I mean the kind of indeterminacy that is characteristic of vague predicates.[2] By *selection indeterminacy* I mean the kind of indeterminacy that is motivated by the problem of the many.[3] It is instructive to investigate how these two kinds of indeterminacy interact with each other, how they differ—in the sense that they are independent from each other—and in particular a problem that arises when we recognize that a single predicate can have them both. Throughout the rest of the paper I shall focus in these issues.

I shall assume some familiarity with the supervaluationist framework and with the supervaluationist treatment of vagueness, given how extensive the literature on this topic is.[4] As such, I won't spend too much time characterizing sorites indeterminacy. Sections 2 and 3 are devoted to the supervaluationist approach to the problem of the many. In sections 4 and 5 I explain how to understand selection and sorites indeterminacy when they are had by a single predicate and in what sense these two kinds of indeterminacy are independent from each other. In section 6 I present a problem (Williams, 2006) that arises in the supervaluationist framework when it is recognized that a single predicate can be both sorites and selection indeterminate. The problem, in a nutshell, is that supervaluationism loses its ability to capture our intuitions concerning the presence or absence of sharp cut-offs. Finally, in section 7 I offer a solution to this problem.

---

1 Fine (1975) has a good example of this. In that paper he defines a predicate ('nice') in such a way that it is indeterminate, but not vague.

2 Weatherson (2010) argues that there is more to vagueness than just being susceptible to the sorites paradox. For this reason I don't identify sorites indeterminacy and vagueness.

3 I shall say more about these two kinds of indeterminacy below.

4 For a comprehensive account of supervaluationism see Keefe (2000).

2. The Problem of the Many

There are particles that are clearly part of Kilimanjaro. For example, if we dig deep into the center of the mountain, we will find a bunch of particles that are clearly part of Kilimanjaro. There are particles that are clearly not part of Kilimanjaro as well. The particles that compose my hand are clear examples of that. But there are also particles such that it is not clear whether or not they are part of Kilimanjaro. Think of all the particles that are not far enough from the mountain to count as clearly not part of the mountain, but that are not close enough to be clearly be part of it. This transition is gradual. Thus, the story goes, Kilimanjaro doesn't have sharp boundaries.

It is plausible to think, then, that there are many aggregates of particles that have equal claim to be Kilimanjaro; given that the mountain doesn't have sharp boundaries, there are many collections of particles that have an equal claim to be the mountain. Let's call these collections of particles *mountain candidates*. Some of them will include more particles than others, but all of them will share most of their parts. If we count one of them as a mountain on the basis that it is too mountain-like not to be counted as a mountain, then all the candidates are mountains. But this seems to be absurd. If we exclude one of the candidates on the basis that there are many other candidates that are equally good candidates, we have to rule out all of them, and therefore say that there is no mountain after all. But this is absurd as well. Thus, either there are many mountains or none. This is, in a nutshell, the problem of the many.[5]

There are at least three straightforward ways to approach this problem. The first one (Unger, 1980) is to conclude that there are no mountains—and, of course, if one thinks this way, then one has to deny that there are persons, cats, clouds, and so on, since we can run a problem of the many argument for these objects as well. The second option (Lewis, 1993) is to conclude that there are millions of mountains (persons, cats, clouds, and so on) when we would normally think there is only one. The third—and more reasonable—option is to stick to our

[5] Given that I'm assuming a semantic approach to indeterminacy I take the problem of the many to be semantic, rather than metaphysical. For a criticism of the metaphysical approach to this problem see Lewis (1993).

intuitions and say that Kilimanjaro is a mountain and that it is the only mountain in the vicinity (McGee and McLaughlin, 2000; Weatherson, 2003).

I take it that one should opt for one of the first two options only if there is no way the third option can be worked out in a plausible way. This is so, because the first two options are highly revisionary regarding the truth and falsehood of our everyday assertions. Radical departures from common sense are only justified when there is no plausible way of making sense of what common sense suggests. For example, if we adopt the first option, whenever we say things like 'John is climbing a mountain' we say something false regardless of what John is actually doing—this follows because according to this view there are no mountains. If we opt for the second alternative, whenever we say things like 'John is climbing only one mountain' what we say is literally false, because according to this view there are millions of mountains that John is climbing.

Now let's consider one way in which the third option can be plausibly defended. The supervaluationst approach to the problem of the many promises to account for the problem while preserving the truth of all (or at least most) of the sentences that we normally accept as true.[6] As such, this view opts for the third kind of solution. According to supervaluationism, when we say things like 'there is only one mountain there [pointing to Kilimanjaro]' we speak truthfully. In the next section we will see with some detail how the supervaluationist argues in favor of this view.

3. The Supervaluationist and the Many

According to supervaluationism, natural language predicates are indeterminate because our linguistic conventions in conjunction with the way the world is aren't enough to determine every single case of application (McGee and McLaughlin, 1995). For instance, our linguistic conventions and the way the world is guarantee that the predicate 'bald' applies to a hairless person and clearly doesn't apply to a hairy person. But our linguistic practices and the way the world is aren't enough to

[6] For criticisms of the supervaluationist approach to the problem of the many see Sorensen (2000), McKinnon, (2002), and Schiffer (2000). For a response to those criticisms see McGee and McLaughlin (2000) and Weatherson (2003).

determine its application to every single case—that is why there are borderline cases.

The problem of the many is a clear reflection of something we already know, namely, that our linguistic practices and the way the world is are not enough to determinate every single case of application of a good number of natural language predicates—and singular terms. Since, for example, the predicate `is a mountain' is far from being perfectly precise, it is not a surprise that when confronted with all the mountain candidates it won't pick out one of them as the unique mountain. However, from this we shouldn't conclude that there are no mountains or that there are many where we think there is only one. The supervaluationist insight is that predicates like 'is a mountain' don't need to be perfectly precise for us to be able to assert sentences like 'John is climbing a mountain'.

Let's see how supervaluationism argues in favor of this view. It would be useful to introduce some terminology before we move on. Following, supervaluationism interprets an indeterminate language *L* by assigning to it a set of *admissible models*.[7] An admissible model is a classical model that satisfies all *penumbral connections* and *classificatory constraints*. A penumbral connection is a logical relation between expressions of a given language. For example, in English something cannot be both red and green all over. Thus, an admissible model in which the predicate 'is completely red' applies to an object is a model in which the predicate 'is completely green' doesn't apply to that object.[8] A classificatory constraint is a constraint to the effect that admissible models must conform to the linguistic conventions in play. Thus, if speakers of language *L* use the predicate 'is bald' in such a way that it applies to John, in every admissible model John is bald. Hence, an admissible model of language *L* respects the linguistic conventions concerning the range of application of *L*-predicates.

A *maximal group* of Φ candidates (from now on Φ-group) is a group such that only Φ candidates can be in it and if a Φ candidate is in that group, all and only the Φ candidates that share most of their parts with it are in that group. For example, a particular mountain-group is a group

7 Admissible models do the same kind of work admissible precisifications do in Fine (1975).

8 Another example of a penumbral connection is this: if x is taller than y and y is tall, then x is tall. Every admissible model is such that this penumbral connection is satisfied.

of mountain candidates—think of all the clouds of particles that are Kilimanjaro candidates—such that all of them share most of their parts.

An *exclusion principle* for predicate 'Φ' is a principle to the effect that for each Φ-group, at most one of the members in it is a Φ. A *selection principle* for a predicate 'Φ' is a principle that picks out only one Φ candidate for each Φ-group. If the selection principle picks out an object *o* as a Φ, then none of the other members of the relevant Φ-group is a Φ. This is all the terminology we need for now.

The supervaluationist view is that even if the predicate 'is a mountain' is indeterminate in its application to all the mountain candidates, that doesn't get in the way of sentences like 'Kilimanjaro is a mountain' being true. The way supervaluationism delivers this result is in the expected way. In each admissible model the predicate 'is a mountain' applies to only one mountain candidate for each mountain-group and in most admissible models the predicate applies to different mountain candidates. This is so because each admissible model sharpens the meaning of each predicate by adopting a particular *selection principle*. Given the classificatory constraints, if an admissible model assigns to 'mountain' only one mountain candidate for each mountain-group, that model also assigns the relevant mountain candidate to, for example, the name 'Kilimanjaro'—this is so, since the speakers of English have adopted enough linguistic conventions to determine that 'is a mountain' applies to Kilimanjaro. Given this, the sentence 'Kilimanjaro is a mountain' is true in every admissible model and, therefore, supertrue.

The idea, then, is that regardless of how we make our language completely precise, sentences like 'Kilimanjaro is a mountain' are true. Furthermore, on the supervaluationist account, it is definitely the case that Kilimanjaro is a mountain and that it is the only mountain occupying roughly that region of space. This is so since in each admissible model the predicate picks out only one mountain candidate as a mountain from the relevant mountain-group and that object is assigned to the name 'Kilimanjaro' in that model. What is false according to supervaluationism is that there is something such that it is determinate that it is a mountain and that it is the only mountain occupying roughly that region of space. I will say more about this below.

4. Sorites and Selection Indeterminate Predicates

We have seen that predicates like 'is a mountain', 'is a person', 'is a cloud', 'is a dog', 'is a table' are selection indeterminate, since all of them are susceptible to the problem of the many. It is easy to show that those predicates are also sorites indeterminate. It is easy to build a suitable sorites series for each of these predicates. Let's consider a concrete example. A sorites series for the predicate 'is a table' is such that the first member is a table and the last member is a small chunk of wood, and the only difference between two adjacent members of this series is a single splinter. Thus, the second member of the series is just like the first one except that a splinter has been removed from it. The third member is just like the second one except that it has one less splinter. And off we go down the slippery slope.

Let's say that a predicate is sorites indeterminate just in case it is indeterminate whether it applies to some members of a suitable sorites series and there are no sharp cut-offs between its positive and negative cases and any other semantic category. It's clear, then, that 'is a table' is sorites indeterminate relative to the sorites series just described. The predicate applies to the first members of the series; it doesn't apply to the last ones, and there is a middle range such that it is indeterminate whether the predicate applies to objects in that range—there we find collections of wood such that it is indeterminate whether they are tables. Also, it is plausible to think that there are no sharp cut-offs between the positive and negative cases of application of this predicate relative to this series and any other semantic category. The intuitive reason for this is that a single splinter cannot make the difference between something that is a table and something that isn't.

A predicate like 'is a table' is both sorites and selection indeterminate. It is easy to show that many other predicates are like this. The supervaluationist treatment of sorites indeterminacy (or vagueness, if you like) is a familiar one. It goes without saying that a supervaluationist would try to account for both sorites and selection indeterminacy in a supervaluationist way. Once we recognize that some natural language predicates have these kinds of indeterminacy, interesting and challenging issues arise regarding how they relate to each other. The first issue concerns their independence—in the sense that a predicate can have one kind of indeterminacy without necessarily having the other. The second issues concerns how the presence of selection indeterminacy makes it harder to understand sorites indeterminacy. I will start by addressing the first issue.

5. Independence

That a predicate can be selection indeterminate without being sorites indeterminate can be shown in a more or less straightforward way. In order to make this point it is useful to define the notion of *partial sharpening.*

- Partial sharpening: is a model that satisfies all penumbral connections and classificatory constraints, where the predicates of *L* are more precise without being absolutely precise.

Assume, for example, that Olivia and Eli are borderline tall, and that Olivia is slightly taller than Eli. A partial sharpening can very well be one where Olivia is tall, and it is indeterminate whether Eli is tall. As such, a partial sharpening is not a classical model. This is so because a partial sharpening leaves room for indeterminacy.

This kind of partial model, then, sharpens indeterminate predicates a little bit, without making them completely precise. Now, there are different kinds of partial sharpenings depending on what gets sharpened. Call a predicate *sorites precise* if it is not sorites indeterminate. A predicate is *selection precise* if it is not selection indeterminate. There are partial sharpenings where predicates like 'is a table' are sorites precise but not selection precise. Call such a model a *partial sorites sharpening*. A partial sharpening can also make predicates—and other expressions—selection precise but not sorites precise. Call such sharpening a *partial selection sharpening*.

We can use these definitions to make clear the sense in which sorites and selection indeterminacy are independent from each other. Consider, for example, the predicate 'is a mountain.' As we have seen, this predicate is both sorites and selection indeterminate. Now, consider all the admissible partial sorites sharpenings. In each of these models the predicate draws a sharp line between all the mountain-group candidates and all the other groups. That is to say, on one side of the cut-off there are many things, all of which are neither a mountain candidate nor a mountain, and on the other side of the cut-off, there are many mountain-group candidates all of which have a group member that is determinately a mountain—this is so, because in these models the predicate is selection indeterminate, but not sorites indeterminate. This shows, I take it, that selection-indeterminacy is independent from sorites-indeterminacy, since the former can be present even in the absence of the later. This argument also shows that we can identify and distinguish these two kinds of indeterminacy when they are had by a single expression.

Now let's show that a predicate can be both selection precise and sorites indeterminate. To do this it will be useful to focus on partial selection sharpenings. It is a bit hard to understand what exactly partial selection sharpenings are, for the following reason. If a predicate is sorites indeterminate, then it has (or can have) borderline cases. Now, take one of those borderline cases. It is obvious that the predicate, let's say 'is a mountain', is indeterminate with respect to that object (in the sense that $\neg DMountain(a) \wedge \neg D\neg Mountain(a)$).[9] If this is so, how can a predicate be selection precise but not sorites precise? If 'is a mountain' is selection precise one would expect that, for each mountain-group, 'is a mountain' definitely applies to one of them and it definitely doesn't apply to the rest. But if 'is a mountain' is sorites-indeterminate, then there is a mountain-group such that the predicate is indeterminate in its application to any of them—this is so when the mountain-group is in the borderline area. If this is right, then partial selection sharpenings seem to be incoherent.

However, there is a way of understanding selection precision such that partial selection sharpenings make sense. In order to do this it is useful to appeal to selection principles. These principles select from every Φ-group of candidates a single object among the many. It is because natural language predicates like `is a mountain' are not precise enough to have a selection principle as part of their meaning that the problem of the many arises in the first place. Be that as it may, each admissible model assigns predicates selection principles—that is, way in each admissible model predicates like 'is a mountain' pick out from each mountain-group only one object among the many candidates.

[9] Where '$D\Phi$' gets interpreted as *it is determinate that Φ*. As usual, $D\Phi$ is true in an admissible model M just in case Φ is true in all the admissible models accessible from M.

Now, each partial selection sharpening makes predicates more precise by specifying a selection principle for each of them. This is not to say that if a predicate has a selection principle it automatically draws a sharp cut-off in a sorites series. So, how should we think of borderline cases in partial selection sharpenings? I claim that this is a perfectly legitimate way of doing just that: within the borderline range the predicate picks out only one object from each Φ-group and it is indeterminate whether that object is Φ. Thus, the predicate picks out one among the many in each Φ-group relative to a partial selection sharpening. This is so because that sharpening assigns to Φ a selection principle, but it is still indeterminate whether `Φ' applies to that object, given that relative to that sharpening the predicate is still sorites indeterminate. Furthermore, relative to these sharpenings the predicate definitely doesn't apply to any of the other Φ candidates —simply because the selection principle doesn't pick them out.

6. Selection Indeterminacy Obscures Sorites Indeterminacy

So far it seems that we can get a clear understanding of the differences and independence between the two kinds of indeterminacy under consideration. However, as we shall see in this section, supervaluationism runs into trouble when predicates are both selection and sorites indeterminate. Williams (2006) convincingly argues that if we give a supervaluationist treatment to both vagueness and the problem of the many, the standard supervaluationist explanation of the presence or absence of sharp cut-offs along a sorites series doesn't hold. In what follows I shall explain the problem and then, in the next section, I shall propose a solution.

The supervaluationist diagnosis of the sorites paradox is that the sorites premise ($\forall x(\Phi x \rightarrow \Phi x')$) is superfalse.[10] As it is well known, given classical logic, it follows that:

(a) $\exists x(\Phi x \wedge \neg\Phi x')$

At first sight, (a) seem to imply that 'Φ' is not in fact vague, since apparently (a) implies that there is a sharp cut-off in the series. However,

---

[10] Of course, this formulation of the sorites premises requires that we restrict our domain to the members of the relevant sorites series.

the supervaluationist semantics disagrees. (a) is supertrue not because 'Φ' draws a sharp cut-off somewhere along the sorites series, but because in each admissible model 'Φ' draws such a cut-off at a different location. For this reason, there is no member of the series such that it is determinate that it is a Φ and its successor is not a Φ. Thus, the following is supertrue as well:

(b) $\neg\exists x D(\Phi x \wedge \neg\Phi x')$

What (b)'s supertruth guarantees, according to supervaluationims, is that there is no sharp cut-off between the positive and negative instances of predicate 'Φ'. Hence, according to this view, we are right in thinking that vague predicates do not draw sharp cut-offs, not because (a) is false (since it is supertrue) but because (b) is supertrue (whenever 'Φ' is vague). Thus, (b)'s supertruth tracks, as it were, our intuitions about sharp cut-offs. It seems, then, that sentences like (b) play a crucial role in the supervaluationist framework, not only because the supertruth or superfashood of statements like (b) are used to explain the presence or absence of sharp cut-offs, but because those sentences play a role in the explanation of our intuitions regarding sharp cut-offs.[11]

The challenge presented in is straightforward. The problem goes as follows: The English predicate 'is a mountain' is certainly sorites-indeterminate. If we accept the supervaluationist treatment of the problem of the many then we are committed to (c)'s supertruth:

(c) $\neg\exists x D(\mathit{Mountain}(x))$

This is so because each mountain candidate is a mountain in some admissible models but not in others, and, therefore, it is not determinately a mountain. Now, consider a series of two mountains —or, to be more precise, two mountain-groups. The first member of the series is the mountain-group corresponding to all the Kilimanjaro candidates and the second one is the mountain-group corresponding to all Tinny candidates (Tinny is a small hill). Even if the predicate 'is

[11] I have some reservations about this kind of explanation. However, this kind of explanation is at the heart of supervaluationism, so I will play along with it.

a mountain' is sorites indeterminate, it draws a sharp cut-off relative to this series—but not in a proper sorites series—given that Kilimanjaro is enormous and Tiny is tiny, and these are the only two members of this series. If so, the supervaluationist explanation of sharp cut-offs predicts that (d) is supertrue (assuming, as we should, that quantification is restricted to the members of this series):

(d) $\exists x D(Mountain(x) \wedge \neg Mountain(x'))$

Now, notice that (c) is supertrue even if we restrict quantification to the members of this series, because each Kilimanjaro candidate is not definitely a mountain and each Tiny candidate is definitely not a mountain. Therefore, since (c) is supertrue, (d) is superfalse. Hence, (e) is supertrue:

(e) $\neg\exists x D(Mountain(x) \wedge \neg Mountain(x'))$

At this point the problem becomes clear. The supervaluationist claims that the predicate 'is a mountain' determines a sharp cut-off in this series (the one that only has two mountain-groups). If this is so, (d) should be supertrue, but it is superfalse. It seems like there is a crash in the supervaluationist framework.

As a matter of fact, the problem is a bit more serious. Notice that (e) is supertrue simply because 'is a mountain' is selection indeterminate. As such, any selection indeterminate predicate 'Φ' is such that (f) is supertrue (provided that we restrict the quantifier to the members a suitable series):

(f) $\neg\exists x D(\Phi x \wedge \neg\Phi x')$

Now, recall that supervaluationism appeals to sentences like (d) to explain our intuition that relative to a suitable sorites series vague predicates do not draw sharp cut-offs. However, we have seen that sentences like (d) are supertrue *independently* of whether 'Φ' draws sharp cut-offs, so long as this predicate is selection indeterminate. The problem is, then, that supervaluationsim doesn't really have a good way of explaining our intuitions regarding sharp cut-offs. When we think of a sorites series for the predicate 'is a mountain' we tend to think that the following English sentence is true: 'There is no member of the series that is a mountain followed by a member that isn't a mountain'. Supervaluationism would like to interpret this sentence as (e). However, as we have seen, the truth conditions of (e) are not quite the truth conditions of the relevant English sentence, because (e) is true independently of whether there are sharp cut-offs—so long as 'is a mountain' is selection indeterminate.[12]

7. Solving the Problem

My diagnosis is that Williams' problem shows that supervaluationism needs more expressive resources than we originally thought.[13] The problem is, as it were, that in the presence of both sorites and selection indeterminacy supervaluationism has trouble tracking our intuitions regarding both kinds of indeterminacy. What we need, then, is to enrich supervaluationism's expressive resources in such a way that it can capture our intuitions regarding cut-offs while having a plausible solution to the problem of the many.

The first thing to note is that supervaluationsim, as it is, has the resources to express the lack of cut-offs between the mountains and the non-mountains in a way that is not clouded by selection indeterminacy. However, the way in which this can be done is too convoluted to keep track of any intuitions we may have regarding these cases. However, as we shall see, we can introduce new expressive resources into the supervaluationist framework that can help solve this problem.

First let's see how supervaluationism can express the absence of cut-offs in a way that is not clouded by selection indeterminacy. In order to

---

12 Williams' approach to this problem is to adopt Lewis' (1993) account of the problem of the many. According to Lewis, strictly speaking there are billions of mountains where we normally think there is only one. However, he argues that if we count by what he calls *almost identity* we can say that there is only one mountain—since all the Kilimanjaro candidates share almost all their parts. If we accept Lewis' solution to the problem of the many we have to say that (c) is superfalse, since each mountain candidate is a mountain in every admissible model. I think this is an extreme reaction to the problem presented in this section. In section 7 I shall argue that there is a way of solving the problem without saying that strictly speaking there are billions of mountains where we ordinarily think there is only one.

13 Thanks to Carlotta Pavese and Agustín Rayo for their comments regarding this section.

do so, we have to clarify certain things. If we ignore the problem of the many, a sorites series looks like this (of course, a real sorites would have more members):

$$0_0 - 0_1 - 0_2 - 0_3 - 0_4 - 0_5 - 0_6 - 0_7 - 0_8 - 0_9 - 0_{10}$$

Where, say, 0_0 is Kilimanjaro and 0_{10} is Tiny. Now, once we recognize the problem of the many, it is better to represent the series like this :

$$0_0^1, 0_0^2, 0_0^3 - 0_1^1, 0_1^2, 0_1^3 - 0_2^1, 0_2^2, 0_2^3 - \ldots - 0_9^1, 0_9^2, 0_9^3 - 0_{10}^1, 0_{10}^2, 0_{10}^3$$

Each horizontal line separates Φ-groups. The 0s that share the same subscript belong to the same Φ-group and each superscript distinguishes members of the same Φ-group. Thus, for example, if we interpret 'Φ' as 'is a mountain', then we can think of 0_0^1, 0_0^2, 0_0^3 as all the Kilimanjaro candidates, 0_1^1, 0_1^2, 0_1^3 as all the Mount Everest candidates, 0_{10}^1, 0_{10}^2, 0_{10}^3 as all the Tiny candidates, etcetera.[14] For each mountain-group there is a set that has as members all and only the elements of that mountain-group.[15] Thus, relative to the series above, the set A_0 has as members only 0_0^1, 0_0^2, 0_0^3, A_1 has as members only 0_1^1, 0_1^2, 0_1^3 etcetera. We say that a Φ-group corresponds to a set A_n if and only if all and only the objects in the Φ-group are members ofA_n. From now on expressions like 'A_n' are only used to denote sets whose only members are all the objects in a Φ-group. When convenient I shall use 'A_n^Φ' to specify that the elements of $A_n$are Φ candidates. Given this, we can also represent a sorites series for 'Φ' as follows:

$$A_0^\Phi - A_1^\Phi - A_2^\Phi - A_3^\Phi - A_4^\Phi - A_5^\Phi - A_6^\Phi - A_7^\Phi - A_8^\Phi - A_9^\Phi - A_{10}^\Phi$$

Similarly, we can represent the series containing only Kilimanjaro and Tiny as follows:

$$A_0^\Phi - A_{10}^\Phi$$

14 This, of course, is an idealization. The number of Kilimanjaro and Tiny candidates is much, much higher.

15 Here is another idealization point. Presumably it is vague how many mountain-candidates there are. So it isn't clear that one can simply talk about *all* the Kilimanjaro candidates.

Where A_0^Φ is the set of Kilimanjaro candidates, A_{10}^Φ is the set of Tiny candidates, and \`Φ' stands for \`is a mountain'. We can use these definitions to solve the problem that William poses, but first we need to make a few observations.

As we have already noted, the root of the problem is that when supervaluationism recognizes that Φ is selection indeterminate, then sentences like (c)—$\neg\exists xD\Phi(x)$—are automatically supertrue. Regardless of how huge Kilimanjaro is, $\exists xDMountain(x)$ is superfalse. This isn't because Kilimanjaro isn't big enough to count as a mountain, it is so because there isn't a Kilimanjaro candidate that is determinately a mountain. Notice, however, that there are sentences similar to $\exists xD\Phi(x)$ that are supertrue. Here is one:

(g) $\exists xD((x \in A_n^\Phi) \wedge \exists y(y \in A_n^\Phi \wedge \Phi(y)))$

(g) is supertrue just in case there is a member x of A_n^Φ such that in every admissible model there is a member of A_n^Φ that is Φ. If we assume, for the sake of simplicity, that the members of A_n^Φ are constant across admissible models, then the supertruth of (g) amounts to this: in each admissible model there is a member of A_n^Φ that is Φ. As such, if we interpret 'Φ' as 'is a mountain', (g)'s truth requires that there is something that is in a group of things all of which have what it takes to be a mountain.

Now let's see how supervaluationism can express, in a general way, the proposition that there is a sharp cut-off relative to the Kilimanjaro-Tiny series. Here is one way of doing just that:

(h) $\exists xD((x \in A_n^\Phi) \wedge \exists y(y \in A_n^\Phi \wedge \Phi(y)) \wedge \neg\exists z(z \in A_{n'}^\Phi \wedge \Phi(z)))$

(h) is a bit complex, but its intuitive meaning is this: there is a member x of A_n^Φ such that in every admissible model there is a member of A_n^Φ (possibly different from x) that is Φ and no member of the successor of A_n^Φ is Φ. Even more intuitively, (g) holds because in each admissible model there is a member of A_0^Φ (the Kilimanjaro candidates set) that is Φ and no member of A_{10}^Φ is Φ in any admissible model. Relative to our restricted domain, (h) is true because all the Kilimanjaro candidates have what it takes to be a mountain in some admissible model and none of the Tiny candidates have what it takes to be a mountain in an admissible model. Given this, (h) captures very well the thought that there is a sharp cut-off relative to the series containing only Kilimanjaro and Tiny.

Here, then, is how supervaluationism can claim that, relative to a suitable sorites series for ′is a mountain′, there are no sharp cut-offs between the mountains and the things that are not mountains (where ′Φ′ is interpreted as ′is a mountain′):

(i) $\neg\exists x D((x \in A_n^{\Phi}) \wedge \exists y(y \in A_n^{\Phi} \wedge \Phi(y)) \wedge \neg\exists z(z \in A_{n'}^{\Phi} \wedge \Phi(z)))$

(i) is, of course, the negation of (h).[16] For (h) to be true it is required that there is no mountain-group (A_n^{Φ}) that in every admissible model has a member that is a mountain, and that no member of its successor ($A_{n'}^{\Phi}$) is a mountain. Thus, the truth of (i) requires that there is no adjacent pair of Φ-groups such that in every admissible model the members of the first one (A_n^{Φ}) have what it takes to be a mountain, and the members of the second one ($A_{n'}^{\Phi}$) are not mountains in any admissible model.[17]

Now, when confronted with a sorites series for predicate ′Φ′ we tend to think that there is no member of the series that is ′Φ′ followed by one that is not ′Φ′. Most philosophers would think that this is a non-negotiable true intuition that must be captured. Similarly, we think, truthfully, that relative to the Kilimanjaro-Tiny series there is a member of the series that is a mountain followed by one that isn′t a mountain. We have seen that it isn′t obvious how supervaluationism can capture these intuitions. Even if sentences like (i) and (h) get the truth conditions right, they are too complex to play this role.

In order to solve the problem we can enrich supervaluationsm by adding two relations: ‘•’ and ‘∘’. These relations are, syntactically, just like the successor function ‘′’ (where ‘α’ is either a proper name or an individual variable and s is a variable assignment function):

[16] Notice that $\exists x D((x \in A_n^{\Phi}) \wedge \exists y(y \in A_n^{\Phi} \wedge \Phi(y)) \wedge \neg\exists z(z \in A_{n'}^{\Phi} \wedge \Phi(z)))$ is supertrue, even relative to suitable sorites series for Φ. This is so because every admissible model is classical. The fact that in (i) there is a quantifier with wide scope over *D* makes all the difference. The effect of this quantifier is that it locks in a pair of Φ-groups that we evaluate from model to model.

[17] Recall that in order to make things simple we are assuming that the members of each Φ-group remain constant across models.

- $M, s \vDash \Phi\alpha^{\bullet}$ iff there is a A_n^{Φ} in M such that $[\![\alpha]\!]_s \in A_n^{\Phi}$ and there is a $x \in A_n^{\Phi}$ such that x is Φ.
- $M, s \vDash \Phi\alpha^{\circ}$ iff there is a A_n^{Φ} in M such that $[\![\alpha]\!]_s \in A_n^{\Phi}$ and there is a $x \in A_{n'}^{\Phi}$ such that x is Φ.

Notice that for '$\Phi\alpha^{\bullet}$' to be true at a model it is only required that the referent of 'α' in that model is in a Φ-group with something that is a Φ. For '$\Phi\alpha^{\circ}$' to be true at a model, on the other hand, it is only required that there is a x that is Φ in the successor of the Φ-group α is a member of.

Equipped with these new expressions we can express (h) and (i) in a more compact way as follows:

(j) $\exists x D(\Phi x^{\bullet} \wedge \neg\Phi x^{\circ})$

(k) $\neg\exists x D(\Phi x^{\bullet} \wedge \neg\Phi x^{\circ})$

The supervaluationist could claim, then, that (j) and (k) track our intuitions about cut-offs, rather than (h) and (i). This kind of explanation is in line with other explanations that supervaluationists (Fine, 1975; McGee and McLaughlin, 1995; Kefee, 2000) are quick to adopt.

Now, one should be careful about how '$^{\bullet}$' gets used. This is so because if we count mountains using sentences like '$\Phi\alpha^{\bullet}$' we can end up concluding, for example, that all the Kilimanjaro candidates are mountains. The argument is quite simple. Let's assign names to all the Kilimanjaro candidates. We can call one of them '1', another one '2', another one '3', and so on. Now, notice that each of these sentences is supertrue:

- $Mountain(1^{\bullet})$
- $Mountain(2^{\bullet})$
- $Mountain(3^{\bullet})$
- etc...

However, we don't want to say that there are as many mountains as there are mountain candidates. This worry can be dismissed. '•' is an expression that is useful for the identification of cut-offs (or their absence). It is a mistake to count mountains, or anything else for that matter, using this expression, since from *Mountain*(1•), *Mountain*(2•), and $1 \neq 2$, it doesn't follow that there are two mountains.[18]

References

Barnes, E. (2010). "Ontic Vagueness: A Guide for the Perplexed," *Nous* 44 (4), 601–627.

Fine, K. (1975). "Vagueness, Truth and Logic," *Synthese* 30 (3), 265–300.

Keefe, R. (2000). *Theories of Vagueness,* Cambridge University Press.

Lappin, S. (2000). "An Intensional Parametric Semantics for Vague Quantifiers,"*Linguistics and Philosophy* 23 (6), 599–620.

Lewis, D. (1993). "Many, but Almost One," O*ntology, Causality and Mind*, 23–42.

McGee, V. and B. McLaughlin (1995). "Distinctions Without a Difference," *The Southern Journal of Philosophy* 33 (S1), 203–251.

McGee, V. and B. McLaughlin (2000). "The Lessons of the Many," *Philosophical Topics* 28, 129–151.

McKinnon, N. (2002). "Supervaluations and the Problem of the Many," *The Philosophical Quarterly* 52 (208), 320–339.

Quine, W. V. O. (1960). *Word and Object,* MIT press.

Schiffer, S. (2000). "Vagueness and Partial Belief," *Philosophical Issues* 10 (1), 220–257.

Sorensen, R. (2000). "Direct Reference and Vague Identity," *Philosophical Topics* 28, 175–194.

Sorensen, R. (2001). *Vagueness and Contradiction,* Oxford University Press.

Unger, P. (1980). "The Problem of the Many," *Midwest Studies in Philosophy* 5 (1), 411–468.

Weatherson, B. (2003). "Many Many Problems," *The Philosophical Quarterly* 53 (213), 481–501.

[18] For their many comments I'm grateful to Brian McLaughlin, Carlotta Pavese, and Agustín Rayo.

Weatherson, B. (2010). "Vagueness as Indeterminacy," *Cuts and Clouds: Vaguenesss, its Nature and its Logic*, 77–90.

Williams, J. R. G. (2006). "An Argument for the Many," *Proceedings of the Aristotelian Society (Hardback)*, Volume 106, pp. 411–419. Wiley Online Library.

Williamson, T. (1994). *Vagueness*, Routledge.