

<http://doi.org/10.21555/top.v710.2825>

## Is Szabolcsi's Logic a Fuzzy Logic?

¿Es la lógica de Szabolcsi una lógica difusa?

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Recibido: 12 - 05 - 2023.

Aceptado: 04 - 10 - 2023.

Publicado en línea: 23 - 12 - 2024.

Cómo citar este artículo: Castro-Manzano, J.-M. (2025). ¿Es la lógica de Szabolcsi una lógica difusa? *Tópicos, Revista de Filosofía*, 71, 345-362. DOI: <http://doi.org/10.21555/top.v710.2825>.



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### **Abstract**

In this paper we ask ourselves whether Szabolcsi's numerical term logic is a fuzzy logic. Our answer is in the affirmative. In order to justify such a claim, we first expound some preliminaries that help us understand why the inclusion of fuzzy quantifiers is a sufficient condition for fuzziness. Then we present Szabolcsi's logic, which includes said quantifiers.

*Keywords:* term logic; numerical logic; fuzzy logic.

### **Resumen**

En este artículo nos preguntamos si la lógica de términos numérica de Szabolcsi es una lógica difusa. Nuestra respuesta es afirmativa. Para justificar semejante tesis, primero exponemos unos preliminares para entender por qué la inclusión de cuantificadores difusos es una condición suficiente para considerar a una lógica como lógica difusa. Posteriormente presentamos la lógica de Szabolcsi, la cual incluye tales cuantificadores.

*Palabras clave:* lógica de términos; lógica numérica; lógica difusa.

## 1. Introduction<sup>1</sup>

When it comes to logic, being fuzzy is analogous. On the one hand, the main goal of fuzzy logic is to model vague information, typically by using imprecise predicates (e.g. "Jan is *young*," "Kurt is *bald*," "the water is *cold*," etc.); but on the other, in doing so, it encompasses a variety of semantics (e.g. Łukasiewicz semantics, Gödel semantics, t-norm semantics, etc.) and tools (e.g. fuzzy set theory, fuzzy arithmetic, fuzzy clustering, etc.) that might go beyond the emphasis on predicates. Hence, claiming that some logic or some system is fuzzy depends on various, analogous reasons, one of these being the inclusion of fuzzy or non-crisp quantifiers (e.g. "most," "many," "few," etc.). Granted this proposition, in this short contribution we ask ourselves whether Szabolcsi's logic (Szabolcsi & Englebretsen, 2008)—a term logic *à la* Sommers (1982)—is a fuzzy logic. Our answer is in the affirmative, and our argument goes as follows:

1. A logic is a fuzzy logic if it includes fuzzy quantifiers.
  2. Szabolcsi's logic includes fuzzy quantifiers.
- ∴ Szabolcsi's logic is a fuzzy logic.

In order to support the first premise, I expound some preliminaries that help us understand why the inclusion of fuzzy or non-crisp quantifiers is, in this analogous context, a sufficient condition for fuzzyness (§ 2). Then, in order to unpack the meaning of the second premise, I present Szabolcsi's logic in broad terms (§ 3).

If the previous argument is sound, it would offer a good reason to pay more attention to Szabolcsi's logic, which would be a fuzzy term logic. One of a kind.

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<sup>1</sup> This paper draws on ideas and excerpts from Castro-Manzano (2019, 2021 & 2022).

## 2. Preliminaries

Fuzzy syllogistics is the arrangement of syllogistics (see Appendix A) with fuzzy devices, something that can be done in several ways. One of these ways consists in extending the set of traditional quantifiers (i.e. “all” and “some”) as to include a variety of fuzzy quantifiers, such as absolute quantifiers (e.g. “around,” “more than,” etc.), proportional quantifiers (e.g. “most,” “almost all,” “few,” etc.), quantifiers of exception (e.g. “all but,” “at most,” etc.), comparative quantifiers (e.g. “ $n$  more than,” etc.), proportional comparative quantifiers (e.g. “there are double... than...,” etc.), and similarity quantifiers (e.g. “ $a$  and  $b$  are similar”, etc.) (Pereira-Fariña et al., 2014).

Given this general premise, we can pinpoint different proposals that have included some subset of said quantifiers. The proposal by Dubois & Prade (1988) and Dubois et al. (1993) for example, makes use of fuzzy quantifiers with exact boundaries (e.g. “between 30% and 50%”), exact values (e.g. “100%”), and imprecise boundaries (e.g. “most”). Zadeh’s program defines two types of linguistic quantifiers, absolute (e.g. “almost  $n$ ,” etc.) and proportional (e.g. “more than half,” etc.), and assigns them fuzzy numbers (Zadeh, 1985). Peterson’s project requires the extension of intermediate quantifiers such as “most,” “many,” and “few” (Peterson, 1979). Thompson’s system is similar to Peterson’s, but also includes statistical quantifiers (Thompson, 1982 & 1986). And, finally, Murphree’s system tweaks Sommers’ term logic (Sommers, 1982) in order to model numerical statements (Murphree, 1998).

There are many more systems and proposals, of course—(Pereira-Fariña et al., 2014) provides an excellent overview—but, in the interest of time, that does it for our current purposes. Something we can learn from these preliminaries is that the inclusion of fuzzy quantifiers is a sufficient condition for granting a logic the status of being fuzzy. Indeed, if these systems and proposals can be considered *bona fide* fuzzy logics, and they are regarded as so in virtue of including non-crisp quantifiers, then, surely, for any given logic it suffices to include non-crisp quantifiers to be regarded as a fuzzy logic.

Our next step consists in showing how Szabolcsi’s logic includes this sort of quantifiers.

### 3. Szabolcsi's logic

Szabolcsi's logic (also known as NTL, for Numerical Term Logic) is a conservative extension of Sommers and Englebretsen's logic (see Appendix B) that tries to capture numeracy by representing and performing inference with numerical quantifiers (Szabolcsi & Englebretsen, 2008). In order to display this logic, we will show its syntax and its deductive base.

#### 3.1. Syntax

Since NTL is a conservative extension of TFL, in NTL a basic statement has the following form:

$$\pm_n S \pm P$$

where  $\pm$  is shorthand for the  $+$  and  $-$  functors,  $n \in \mathbb{R}^+$ , and  $S$  and  $P$  are term-schemes. With these components we can represent different sorts of fuzzy quantifiers: traditional, generalized, exact, comparative, fractional, and subjective.

##### 3.1.1. Traditional quantifiers

Following the previous order, let us begin with the representation of traditional quantifiers, which look as follows:

All  $S$  is  $P \equiv$  At most  $0$   $S$  are not  $P \equiv$  At least all but  $0$   $S$  are  
 $P := -_0 S \pm P$

No  $S$  is  $P \equiv$  At most  $0$   $S$  are  $S \equiv$  At least all but  $0$   $S$  are not  
 $P := -_0 S - P$

Some  $S$  is  $P \equiv$  More than  $0$   $S$  are  $P \equiv$  Fewer than all but  $0$   
 $S$  are not  $P := +_0 S \pm P$

Some  $S$  is not  $P \equiv$  More than  $0$   $S$  are not  $P \equiv$  Fewer than  
 all but  $0$   $S$  are  $P := +_0 S - P$

Notice that besides the canonical form of a statement (the leftmost), there are a couple of additional representations. So, for instance, when we say, “All logicians are rational,” that is equivalent ( $\equiv$ ) to “At most 0 logicians are not rational” and “At least all but 0 logicians are rational.”

### 3.1.2. Generalized quantifiers

Something similar happens with the generalization of these statements for  $n > 0$ , as follows:

At most  $n$   $S$  are not  $P \equiv$  At least all but  $n$   $S$  are  $P := -_n S+P$

At most  $n$   $S$  are  $P \equiv$  At least all but  $n$   $S$  are not  $P := -_n S-P$

More than  $n$   $S$  are  $P \equiv$  Fewer than all but  $n$   $S$  are not  $P := +_n S+P$

More than  $n$   $S$  are not  $P \equiv$  Fewer than all but  $n$   $S$  are  $P := +_n S-P$

As with traditional quantifiers, the first expressions are called simple representations, whereas the second expressions are known as exceptive representations. For example, “At most 5 logicians are rational” is equivalent to “At least all but 5 logicians are not rational.”

### 3.1.3. Exact quantifiers

These last representations are important because they help us depict exact quantifiers as follows:

Exactly  $n$   $S$  are  $P := +_{(n-1)} S+P + (-_n S+P)$

Exactly  $n$   $S$  are not  $P := +_{(n-1)} S-P + (+_n S+P)$

So, for example, to represent the claim that there are exactly two logicians that are rational we would write  $+_{(1)} L+R + (-_2 L+R)$ , that is to say, more than one logician is rational and at most 2 logicians are rational.

### 3.1.4. Comparative quantifiers

Now, given this account of exact quantifiers, comparative quantifiers require some additional tweaks:

More S than M are P :=  $+(+_{n-1}S+P)+(-_nS-P)+((+_{m-1}M+P)+(-_mM-P))_{m>n}$

Fewer S than M are P :=  $+(+_{n-1}S+P)+(-_nS-P)+((+_{m-1}M+P)+(-_mM-P))_{m<n}$

### 3.1.5. Fractional quantifiers

Fractional quantifiers can also be formalized as follows:

At most  $n/m$  of S are (not) P :=  $-_{n/m}S\pm P$

More than  $n/m$  of S are (not) P :=  $+_{n/m}S\pm P$

At least  $n/m$  of all S are (not) P :=  $+_{n/m-1}S\pm P$

Fewer than  $n/m$  of all S are (not) P :=  $-_{n/m-1}S\pm P$

### 3.1.6. Subjective quantifiers

And finally, with some other tweaks, we can model subjective quantifiers as follows:

Many S are (not) P :=  $+_qS\pm P$

Many more than  $n$  S are (not) P :=  $+_{n+q}S\pm P$

Many fewer than  $n$  S are (not) P :=  $-_{n+q^*}S\pm P$

Most S are (not) P :=  $+(-_mS\pm P)+(m<ts/2)$

Few S are (not) P :=  $+(-_fS\pm P)+(f<ts/2)$

Enough S are (not) P :=  $+_{e^*}S\pm P$

More than enough S are (not) P :=  $+_eS\pm P$

Exactly enough S are P :=  $+(+_{e^*}S+P)+(-_eS-P)$

Exactly enough S are not P :=  $+(+_{e^*}S-P)+(-_eS+P)$

Too many S are (not) P :=  $+_vS\pm P$

Almost  $n$  S are P :=  $+(+_{n-a} S+P)+(-_{n-a} S-P)$

Almost  $n$  S are not P :=  $+(+_{n-a} S-P)+(-_{n-a} S+P)$

Just over  $n$  S are P :=  $+(+_{n+w} S+P)+(-_{n+w} S-P)$

Just over  $n$  S are not P :=  $+(+_{n+w} S-P)+(-_{n+w} S+P)$

And that does it for representation. What we can gather, then, is that NTL does include fuzzy quantifiers from the standpoint of syntax; however, that is not a sufficient condition for NTL to be regarded as a fuzzy logic, let alone a logic. In order for NTL to be considered as a *bona fide* logic, we need a notion of validity. And Szabolcsi provided it.

### 3.2. Validity

Again, since NTL is a conservative extension of TFL, we say an inference is valid (in NTL) iff:

1. The algebraic sum of the premises is equal to the conclusion;
2. the number of particular conclusions (*viz.*, zero or one) is equal to the number of particular premises; and
3. the sum of the numerical values of the premises are equal to or greater than the numerical quantifier of the conclusion (this is called quantifier addition, QA).

Put like this, inference in NTL seems too simple to be sound or even interesting; but that would be a hasty claim. In order to further explain the deductive power of this system, we need to deploy some logically previous concepts (namely, quantifier transformation, guaranteed reference, and term distribution), and some examples.

#### 3.2.1. Quantifier transformation

In NTL, terms represent sets, and so any numerically quantified statement makes an assertion with respect to the size of the set. Take for



example the statement  $-_{10}S+P$ . If we let  $|S|$  stand for the cardinality of  $S$ , the sentence  $-_{10}S+P$  is equivalent to  $+_{|S|-10^*}S+P$ , so that the following hold:

$$+_{n}S\pm P \equiv -_{|S|-n^*}S\pm P$$

$$+_{n^*}S\pm P \equiv -_{|S|-n}S\pm P$$

$$-_{n}S\pm P \equiv +_{|S|-n^*}S\pm P$$

$$-_{n^*}S\pm P \equiv +_{|S|-n}S\pm P$$

The expressions to the left are called numerically simple, while the expressions to the right are called numerically exceptive. The process by which we convert a numerically simple expression into a numerically exceptive expression is called quantifier transformation (*QT*).

In order to accomplish such a transformation, Szabolcsi proposes the next steps: change the quantifier, i.e. substitute the initial + (resp. -) for - (resp. +); change the numerical interpretation from simplicity to exceptivity (or viceversa); and add 1 to a quantity that is affixed with an \* (the star, \*, represents an exclusive lower limit: for instance, in  $+_{n^*}S+P$ ,  $n^*=n-1$ ). So, for example, the following statements are equivalent in virtue of quantifier transformation: "All but 15 logicians are democrats" (i.e.  $-_{15}L+D$ ) is the same as "At least  $|L|-15$  logicians are democrats (i.e.  $+_{|L|-15^*}L+D$ ).

### 3.2.2. Guaranteed reference

Given any statement in NTL, the guaranteed reference (or gr-value) of such statement with respect to a subset of its terms is the value obtained by reducing it to its standard form. For example, say we have the statement "At least 3 logicians are not republicans," namely,  $+_{3}L-R$ , then its reduction to standard form would be  $+_{4^*}L-R$ , and hence its gr-value would be 4 with respect to the set  $R$ .

### 3.2.3. Term distribution

When the domain of a term  $T$  has a gr-value  $|T|-n$  with respect to some subset  $T$ , then  $T$  is said to be distributed within the domain; otherwise,  $T$  is undistributed within such domain. In general, a term is distributed in a domain if and only if the domain has a gr-value to at least all but some excepted number of members of the set that the term

represents. For example, in the statement “All logicians are rational,” the term “logicians” is distributed whereas the term “rational” is not, for the statement  $-L+R$  can be reduced to its standard form  $+_{|L|-0}L+R$ , whose domain has a gr-value with respect to the subset that results from the intersection of L and R, where this value contains a variable representing the total number of members of the set it represents, but there is no such number for the term R.

With these provisions, quantifier addition (QA) can be reformulated as follows: if a term T occurs distributed in a statement, wherein its domain D has a gr-value of  $|T|-n$  with respect to a subset T, and T occurs undistributed in another statement, wherein its domain D' has a gr-value of m with respect to a subset T and  $m>n$ , then we can infer a new statement that is exactly like the second, except that T has been replaced by the first statement minus its distributed T, and wherein the domain D' has a gr-value that is equal to  $m-n$  with respect to some subset; otherwise, the inference is not valid.

### 3.3. Examples

*Longum iter est per praecepta*, so let us show some examples to get a sense of the power of NTL. Here are some examples regarding the different sorts of quantifiers NTL is able to model (tables 1-8).

| Statement      | NTL      |        |
|----------------|----------|--------|
| 1. All M are P | $-_0M+P$ | P      |
| 2. All S are M | $-_0S+M$ | P      |
| ∴ All S are P  | $-_0S+P$ | QA 1,2 |

Table 1. A valid NTL inference using traditional quantifiers

| Statement              | NTL         |        |
|------------------------|-------------|--------|
| 1. All but 7 M are P   | $-_7M+P$    | P      |
| 2. At least 30 S are M | $+_{30}S+M$ | P      |
| ∴ At least 23 S are P  | $+_{23}S+P$ | QA 1,2 |

Table 2. A valid NTL inference using generalized quantifiers

| Statement                       | NTL         |          |
|---------------------------------|-------------|----------|
| 1. All but 10 M are P           | $-_{10}M+P$ | <i>P</i> |
| 2. All but 5 S are M.           | $-_5S+M$    | <i>P</i> |
| $\therefore$ All but 15 S are P | $-_{15}S+P$ | QA 1,2   |

Table 3. Another valid NTL inference using generalized quantifiers

| Statement  | NTL                   |               |
|--|-----------------------|---------------|
| 1. At least all but 3 A have more than 18 C            | $-_3A+(+H+_{18}C)$    | <i>P</i>      |
| 2. At most 8 F are not A                               | $-_8F+A$              | <i>P</i>      |
| 3. Exactly 6 C are not L                               | $+(+_6C-L)+(-_6C+L)$  | <i>P</i>      |
| $\therefore$ At least all but 11 F have more than 12 L | $+_{11}F+(+H+_{12}L)$ |               |
| 4.   | $-_{11}F+(+H+_{18}C)$ | QA 1,2        |
| 5.   | $-_6C+L$              | <i>Simp</i> 3 |
| 6.   | $+_{11}F+(+H+_{12}L)$ | QA 4,5        |

Table 4. A valid NTL inference using exact quantifiers

| Statement                               | NTL                        |                  |
|---|----------------------------|------------------|
| 1. There are more S in P than in B      | $ SP  >  SB $              | <i>P</i>         |
| 2. There are fewer P who S than P who C | $ SP  <  CP $              | <i>P</i>         |
| 3. 2 million in B are also S            | $ BS  =  SB  = 2000000$    | <i>P</i>         |
| 4. There are 10 million in P            | $ P  = 10000000$           | <i>P</i>         |
| ∴ Fewer than 8 million in S is also C   | $-_{8000000} P+C$          |                  |
| 5.                                      | $ CP  >  SB $              | From 1,2         |
| 6.                                      | $ CP  > 2000000$           | From 3           |
| 7.                                      | $+_{2000000} C+P$          | QA 4,5           |
| 8.                                      | $+_{2000000} P+C$          | Com 7            |
| 9.                                      | $+_{ P -2000000} P+C$      | QT 8             |
| 10.                                     | $-_{10000000-2000000} P+C$ | Substitution 4,9 |
| 11.                                     | $-_{8000000} P+C$          | From 10          |

Table 5. A valid NTL inference using comparative quantifiers

| Statement                      | NTL                  |          |
|--------------------------------|----------------------|----------|
| 1. More than 73% of P are D    | $+_{ P-73/100 } P+D$ | <i>P</i> |
| 2. Everyone who is D is also S | $-_0 D+S$            | <i>P</i> |
| ∴ Fewer than 27% P are not S   | $+_{ 27/100 } P+S$   |          |
| 3.                             | $+_{ P-73/100 } P+D$ | QT 1     |
| 4.                             | $+_{ 27/100 } P+D$   | From 3   |
| 5.                             | $+_{ 27/100 } P+S$   | QA 2,4   |

Table 6. A valid NTL inference using fractional quantifiers

| Statement       | NTL       |               |
|-----------------|-----------|---------------|
| 1. Most P are M | $-_m P+M$ | <i>P</i>      |
| 2. All M are S  | $-_q M+S$ | <i>P</i>      |
| ∴ Most P are S  | $-_m P+S$ | <i>QA 1,2</i> |

Table 7. A valid NTL inference using subjective quantifiers

| Statement                | NTL               |               |
|--------------------------|-------------------|---------------|
| 1. Many T are L to all S | $-_q T+(+L-S)$    | <i>P</i>      |
| 2. Most A are S          | $-_m A+S$         | <i>P</i>      |
| ∴ Most A are L to many T | $-_m A+(+L+_q T)$ |               |
| 3.                       | $-_m A+(+_q T+L)$ | <i>QA 1,2</i> |
| 4.                       | $-_m A+(+L+_q T)$ | <i>Com 3</i>  |

Table 8. Another valid NTL inference using subjective quantifiers

#### 4. Final remarks

In this short contribution, we asked ourselves whether Szabolcsi's numerical term logic was a fuzzy logic. Our argument rested, broadly, on the assumption that the inclusion of fuzzy quantifiers is a sufficient condition for fuzzyness, and so we presented Szabolcsi's logic, which includes said quantifiers. The job is done. However, at this point, someone might wonder why bother doing such a job, why asking whether Szabolcsi's logic is a fuzzy logic, and the answer is rather simple: we asked the previous question because symbols matter, and dates are symbolic. About twenty years ago, shortly after finishing his work, Lorne Szabolcsi (1974-2002) passed away. Had he still been with us, who knows what wonderful directions his logic would have taken, specially within the context of revival of term logics (Sommers, 1982; Englebretsen, 1996; Wang, 1997; Correia, 2017; Simons, 2020).

This short contribution pays homage to Szabolcsi's logic. We hope this helps attract more attention to his work. It surely deserves it.

## Appendix A

Syllogistics is a term logic that has its origins in Aristotle's *Prior Analytics* and deals with inference between categorical statements. A categorical statement is a statement composed of two terms, a quantity, and a quality. The subject and the predicate of a statement are called terms: the term-schema S denotes the subject term of the statement and the term-schema P denotes the predicate. The quantity may be either universal (*All*) or particular (*Some*) and the quality may be either affirmative (*is*) or negative (*is not*). These categorical statements have a type denoted by a label—either a (universal affirmative, SaP), e (universal negative, SeP), i (particular affirmative, SiP), or o (particular negative, SoP)—that allows us to determine a mood, that is, a sequence of three categorical statements ordered in such a way that two statements are premises (major and minor) and the last one is a conclusion. A categorical syllogism, then, is a mood with three terms, one of which appears in both premises but not in the conclusion. This particular term, usually denoted with the term-schema M, works as a link between the remaining terms and is known as the middle term. According to the position of this middle term, four figures can be set up in order to encode the valid syllogistic moods. For the sake of brevity—but without loss of generality—we have omitted the syllogisms that require existential import (table A1).

| Figure 1 | Figure 2 | Figure 3 | Figure 4 |
|----------|----------|----------|----------|
| aaa      | eae      | iai      | aee      |
| eae      | aee      | a ii     | iai      |
| a ii     | eio      | oao      | eio      |
| eio      | aoo      | eio      |          |

Table A1. Valid syllogistic moods

## Appendix B

Term Functor Logic (TFL) is a plus-minus term logic in which a categorical statement is a statement of the form  $\pm S \pm P$  where  $\pm$  is shorthand

for the + and - functors, and S and P are term-schemes. Given this language, TFL offers a basic sense of validity as follows (Englebretsen, 1996, p. 167): a syllogism is valid iff i) the algebraic sum of the premises is equal to the conclusion, and ii) the number of particular conclusions (*viz.*, zero or one) is equal to the number of particular premises (this rule is also known as the *dictum de omni et nullo* or *DON*). And so, with this logic, we can model assertoric inferences like the one shown in table A2.

| Statement      | TFL  |         |
|----------------|------|---------|
| 1. All M are P | -M+P | P       |
| 2. All S are M | -S+M | P       |
| ∴ All S are P  | -S+P | DON 1,2 |

Table A2. A valid TFL inference

That does it for syllogistic inference, but TFL includes more rules, such as:

1. Premise (*P*): Any premise or tautology can be entered as a line in proof. (Tautologies that repeat the corresponding conditional of the inference are excluded. The corresponding conditional of an inference is simply a conditional sentence whose antecedent is the conjunction of the premises and whose consequent is the conclusion.)
2. Double Negation (*DN*): Pairs of unary minuses can be added or deleted from a formula (i.e.  $--X = X$ ).
3. External Negation (*EN*): An external unary minus can be distributed into or out of any phrase:  
 $-(\pm X \pm Y) = \mp X \mp Y$ .
4. Internal Negation (*IN*): A negative qualifier can be distributed into or out of any predicate-term:  $\pm X - (\pm Y) = \pm X + (\pm Y)$ .

5. Commutation (*Com*): The binary plus is symmetric (i.e.  $+X+Y = +Y+X$ ).
6. Association (*Assoc*): The binary plus is associative:  $+X+(+Y+Z) = +(X+Y)+Z$ .
7. Contraposition (*Contrap*): The subject- and predicate-terms of a universal affirmation can be negated and can exchange places:  
 $-X+Y = -(-Y)+(-X)$ .
8. Predicate Distribution (*PD*): A universal subject can be distributed into or out of a conjunctive predicate:  $-X+(+Y+Z) = +(-X+Y)+(-X+Z)$ ; and a particular subject can be distributed into or out of a disjunctive predicate:  $+X+(-(-Y)-(-Z)) = --(-X+Y)-(-X+Z)$ .
9. Iteration (*It*): The conjunction of any term with itself is equivalent to that term (i.e.  $+X+X = X$ ).
10. *Dictum de omni et nullo* (*DON*): If a term, T, occurs universally quantified in a formula and either T occurs not universally quantified or its logical contrary occurs universally quantified in another formula, deduce a new formula that is exactly like the latter except that T has been replaced at least once by the first formula minus its universally quantified T.
11. Simplification (*Simp*): Either conjunct can be deduced from a conjunctive formula. From a particularly quantified formula with a conjunctive subject-term, deduce either the statement form of the subject-term or a new statement just like the original but without one of the conjuncts of the subject-term; i.e., from  $+(+X+Y)\pm Z$  deduce any of the following:  $+X+Y$ ,  $+X\pm Z$ , or  $+Y\pm Z$ . From a universally quantified formula with a conjunctive predicate-term, deduce a new statement just like the original



but without one of the conjuncts of the predicate-term; i.e., from  $\neg X \pm (+Y+Z)$  deduce either  $\neg X \pm Y$  or  $\neg X \pm Z$ .

12. Addition (*Add*): Any two previous formulae in a sequence can be conjoined to yield a new formula, and from any pair of previous formulae that are both universal affirmations and share a common subject-term a new formula can be derived that is a universal affirmation, has the subject-term of the previous formulae, and has the conjunction of the predicate-terms of the previous formulae as its predicate-term; i.e., from  $\neg X+Y$  and  $\neg X+Z$  deduce  $\neg X+(+Y+Z)$ .

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